TRUNG TÂM GIA SƯ LUYỆN THI ALPHA THÀNH PHỐ VINH

CHUYÊN ĐỀ

PHƯƠNG TRÌNH LƯỢNG GIÁC

KIẾN THỨC CẦN NHỚ

I.CÁC CÔNG THỨC BIẾN ĐỔI LƯỢNG GIÁC

1.CÔNG THỨC CỘNG

$\cos(a + b) = \cos a.\cos b - \sin a.\sin b$

$\cos(a - b) = \cos a.\cos b + \sin a.\sin b$

$\sin(a + b) = \sin a.\cos b + \cos a.\sin b$

$\sin(a - b) = \sin a.\cos b - \cos a.\sin b$

$\tan(a + b) = \dfrac{\tan a + \tan b}{1 - \tan a.\tan b}$

$\tan(a - b) = \dfrac{\tan a - \tan b}{1 + \tan a.\tan b}$

2.CÔNG THỨC NHÂN ĐÔI

$\cos 2a = \cos^2 a - \sin^2 a$

$= 2\cos^2 a - 1$

$= 1 - 2\sin^2 a$

$\sin 2a = 2.\sin a.\cos a$

$\tan 2a = \dfrac{2.\tan a}{1 - \tan^2 a}$

3.CÔNG THỨC HẠ BẬC $\cos^2 a = \dfrac{1 + \cos 2a}{2}$

$\sin^2 a = \dfrac{1 - \cos 2a}{2}$

4.CÔNG THỨC BIẾN ĐỔI TỔNG THÀNH TÍCH

$\cos a + \cos b = 2.\cos\dfrac{a + b}{2}.\cos\dfrac{a - b}{2}$

$\cos a - \cos b = -2.\sin\dfrac{a + b}{2}.\sin\dfrac{a - b}{2}$

$\sin a + \sin b = 2.\sin\dfrac{a + b}{2}.\cos\dfrac{a - b}{2}$

$\sin a - \sin b = 2.\cos\dfrac{a + b}{2}.\sin\dfrac{a - b}{2}$

$$\tan a + \tan b = \frac{\sin(a+b)}{\text{cosacosb}}$$

$$\tan a - \tan b = \frac{\sin(a-b)}{\text{cosacosb}}$$

5.CÔNG THỨC BIẾN ĐỔI TÍCH THÀNH TỔNG

$$\text{cosa.cosb} = \frac{1}{2}[\cos(a-b) + \cos(a+b)]$$

$$\text{sina.sinb} = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$$

$$\sin a\cos b = \frac{1}{2}\left[\sin(a+b) + \sin(a-b)\right]$$

$$\cos a\sin b = \frac{1}{2}\left[\sin(a+b) - \sin(a-b)\right]$$

6.BẢNG GIÁ TRỊ LƯỢNG GIÁC CỦA CÁC CUNG ĐẶC BIỆT

x	rad	$-\pi$	$-\frac{5\pi}{6}$	$-\frac{3\pi}{4}$	$-\frac{2\pi}{3}$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
	độ	-180^o	-150^o	-135^o	-120^o	-90^o	-60^o	-45^o	-30^o	0	30^o	45^o	60^o	90^o	120^o	135^o	150^o	180^o
sin		0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
cos		-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
tan		0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	||	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	||	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0
cot		||	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	||	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	||

II.CÁC PHƯƠNG TRÌNH LƯỢNG GIÁC THƯỜNG GẶP

1.Phương trình sinx=a.(-1≤ a ≤ 1)

$\sin x = a \Leftrightarrow \left[\begin{array}{l} x = \arcsin a + k2\pi \\ x = \pi - \arcsin a + k2\pi \end{array}\right.; k \in Z$ $\quad +\sin x = \sin\alpha \Leftrightarrow \left[\begin{array}{l} x = \alpha + k2\pi \\ x = \pi - \alpha + k2\pi \end{array}\right.; k \in Z$ ($a = \sin\alpha$)

$\sin x = 0 \Leftrightarrow x = k\pi; k \in Z$

$\sin x = 1 \Leftrightarrow x = \frac{\pi}{2} + k2\pi; k \in Z$

$\sin x = -1 \Leftrightarrow x = -\frac{\pi}{2} + k2\pi; k \in Z$

2.Phương trình cosx=a.(-1≤ a ≤ 1)

$\cos x = a \Leftrightarrow \left[\begin{array}{l} x = \arccos a + k2\pi \\ x = -\arccos a + k2\pi \end{array}\right.; k \in Z$ $\quad +\cos x = \cos\alpha \Leftrightarrow \left[\begin{array}{l} x = \alpha + k2\pi \\ x = -\alpha + k2\pi \end{array}\right.; k \in Z$ ($a = \cos\alpha$)

$\cos x = 0 \Leftrightarrow x = \frac{\pi}{2} + k\pi; k \in Z$

$\cos x = 1 \Leftrightarrow x = k2\pi; k \in Z$

$\cos x = -1 \Leftrightarrow x = \pi + k2\pi; k \in Z$

3.Phương trình tanx=a.

TXĐ: $\mathbb{R} \setminus \left\{\frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\}$

$+\tan x = a \Leftrightarrow x = \arctan a + k\pi, k \in \mathbb{Z}$ $\quad +\tan x = \tan\alpha \Leftrightarrow x = \alpha + k\pi, k \in \mathbb{Z}$

$\tan x = 1 \Leftrightarrow x = \frac{\pi}{4} + k\pi, k \in \mathbb{Z}$

$\tan x = -1 \Leftrightarrow x = -\frac{\pi}{4} + k\pi, k \in \mathbb{Z}$

$\tan x = 0 \Leftrightarrow x = k\pi, k \in \mathbb{Z}$

4.Phương trình cotx=a.

TXĐ: $\mathbb{R} \setminus \{k\pi, k \in \mathbb{Z}\}$

$+\cot x = a \Leftrightarrow x = \text{arccot}\, a + k\pi, k \in \mathbb{Z}$ $\quad +\cot x = \cot\alpha \Leftrightarrow x = \alpha + k\pi, k \in \mathbb{Z}$

$\cot x = 1 \Leftrightarrow x = \frac{\pi}{4} + k\pi, k \in \mathbb{Z}$

$\cot x = -1 \Leftrightarrow x = -\frac{\pi}{4} + k\pi, k \in \mathbb{Z}$

$\cot x = 0 \Leftrightarrow x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$

III.CÁC PHƯƠNG TRÌNH LƯỢNG GIÁC THƯỜNG GẶP.

1.Phương trình a.sinx+bcosx=c ($a^2 + b^2 \neq 0$ **)**

$$\Leftrightarrow \frac{a}{\sqrt{a^2+b^2}}\sin x + \frac{b}{\sqrt{a^2+b^2}}\cos x = \frac{c}{\sqrt{a^2+b^2}}$$

đặt: $\begin{cases} \cos\alpha = \dfrac{a}{\sqrt{a^2+b^2}} \\ \sin\alpha = \dfrac{b}{\sqrt{a^2+b^2}} \end{cases}$

phương trình trở thành: $\sin x\cos\alpha + \cos x \sin\alpha = \frac{c}{\sqrt{a^2+b^2}}$

$$\Leftrightarrow \sin(x+\alpha) = \frac{c}{\sqrt{a^2+b^2}}$$

Chú ý

+Phương trình có nghiệm khi $c^2 \leq a^2 + b^2$

+Nếu $a.b \neq 0, c = 0$ thì: $a\sin x + b\cos x = 0 \Leftrightarrow \tan x = -\frac{b}{a}$

2.Phương trình : $a\sin^2 x + b\sin x\cos x + c\cos^2 x = 0$ **(1)**

+Nếu a = 0: $b\sin x\cos x + c\cos^2 x = 0$

$$\Leftrightarrow \cos x(b\sin x + c\cos x) = 0 \Leftrightarrow \left[\begin{array}{l} \cos x = 0 \\ b\sin x + c\cos x = 0 \end{array}\right.$$

+Nếu c = 0: $a\sin^2 x + b\sin x\cos x = 0$

$$\Leftrightarrow \sin x(a\sin x + b\cos x) = 0 \Leftrightarrow \left[\begin{array}{l} \sin x = 0 \\ a\sin x + b\cos x = 0 \end{array}\right.$$

+Nếu $a \neq 0, c \neq 0, \cos x \neq 0$: (1) $\Leftrightarrow a\frac{\sin^2 x}{\cos^2 x} + b\frac{\sin x\cos x}{\cos^2 x} + c\frac{\cos^2 x}{\cos^2 x} = 0$

$$\Leftrightarrow a\tan^2 x + b\tan x + c = 0$$

BÀI TẬP.

Bài 1. Giải các phương trình:

a) $\sqrt{2}\cot(5x-\frac{\pi}{8})=0$ b) $2\cos^2 x+\sqrt{3}\cos x=0$

c) $\sqrt{3}\sin 3x-\cos 3x=2$ d) $\sin^2 x+\sin 2x+2\cos^2 x=2$

Giải.

a) $\sqrt{2}\cot(5x-\frac{\pi}{8})=0 \Leftrightarrow 5x-\frac{\pi}{8}=\frac{\pi}{2}+k\pi \Leftrightarrow x=\pi+\frac{k\pi}{5}$

b) $2\cos^2 x+\sqrt{3}\cos x=0$

$$\Leftrightarrow \left[\begin{array}{l}\cos x=0\\ \cos x=-\frac{\sqrt{3}}{2}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{2}+k\pi\\ x=\pm\frac{5\pi}{6}+k2\pi\end{array}\right., k\in\mathbb{Z}$$

c) $\sqrt{3}\sin 3x-\cos 3x=2$

$$\Leftrightarrow \frac{\sqrt{3}}{2}\sin 3x-\frac{1}{2}\cos 3x=1 \Leftrightarrow \sin\ (3x-\frac{\pi}{6})=1 \Leftrightarrow 3x-\frac{\pi}{6}=\frac{\pi}{2}+k2\pi \Leftrightarrow x=\frac{2\pi}{9}+\frac{k2\pi}{3}$$

d) $\sin^2 x+\sin 2x+2\cos^2 x=2$

$$\Leftrightarrow \sin x\,(2\cos x-\sin x)=0 \Leftrightarrow \left[\begin{array}{l}\sin x=0\\ \tan x=2\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=k\pi\\ x=\arctan 2+k\pi\end{array}\right.$$

Bài 2. Giải các phương trình:

a) $\sqrt{3}\tan(3x+\frac{3\pi}{5})=0 \Leftrightarrow 3x+\frac{3\pi}{5}=k\pi \Leftrightarrow x=-\frac{\pi}{5}+\frac{k\pi}{3}$

b) $$2\sin^2 x-\sin x-1=0 \Leftrightarrow \left[\begin{array}{l}\sin x=1\\ \sin x=-\frac{1}{2}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{2}+k2\pi\\ x=-\frac{\pi}{6}+k2\pi, k\in\mathbb{Z}\\ x=\frac{7\pi}{6}+k2\pi\end{array}\right.$$

c) $\sin 5x+\cos 5x=-\sqrt{2}$

$$\Leftrightarrow \frac{1}{\sqrt{2}}\sin 5x+\frac{1}{\sqrt{2}}\cos 5x=-1 \Leftrightarrow \sin\ (5x+\frac{\pi}{4})=-1 \Leftrightarrow 5x+\frac{\pi}{4}=-\frac{\pi}{2}+k2\pi \Leftrightarrow x=-\frac{3\pi}{20}+\frac{k2\pi}{5}$$

d) $3\sin^2 x+\sin 2x+\cos^2 x=3$

$$\Leftrightarrow 2\sin x\cos x-2\cos^2 x=0 \Leftrightarrow 2\cos x(\sin x-\cos x)=0$$

$$\Leftrightarrow \left[\begin{array}{l} \cos x=0 \\ \tan x=1 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{2}+k2\pi \\ x=\frac{\pi}{4}+k\pi \end{array}\right.$$

e. $\cos 2x+3\sin x-2=0$

$$\Leftrightarrow 1-2\sin^2 x+3\sin x-2=0 \Leftrightarrow 2\sin^2 x-3\sin x+1=0$$

$$\Leftrightarrow \left[\begin{array}{l} \sin x=1 \\ \sin x=\frac{1}{2} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{2}+k2\pi \\ x=\frac{\pi}{6}+k2\pi \ ,k\in\mathbb{Z} \\ x=\frac{5\pi}{6}+k2\pi \end{array}\right.$$

f. $\sqrt{3}\sin x+\cos x=\sqrt{2} \Leftrightarrow \frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\cos x=\frac{\sqrt{2}}{2}$

$$\Leftrightarrow \sin x\cos\frac{\pi}{6}+\cos x\sin\frac{\pi}{6}=\frac{\sqrt{2}}{2} \Leftrightarrow \sin(x+\frac{\pi}{6})=\sin\frac{\pi}{4}$$

$$\Leftrightarrow \left[\begin{array}{l} x+\frac{\pi}{6}=\frac{\pi}{4}+k2\pi \\ x+\frac{\pi}{6}=\frac{3\pi}{4}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{12}+k2\pi \\ x=\frac{7\pi}{12}+k2\pi \end{array}\right. ,k\in\mathbb{Z}$$

g. $\sqrt{3}\sin x-\cos x=\sqrt{2} \Leftrightarrow \frac{\sqrt{3}}{2}\sin x-\frac{1}{2}\cos x=\frac{\sqrt{2}}{2}$

$$\Leftrightarrow \sin x\cos\frac{\pi}{6}-\cos x\sin\frac{\pi}{6}=\frac{\sqrt{2}}{2} \Leftrightarrow \sin(x-\frac{\pi}{6})=\sin\frac{\pi}{4}$$

$$\Leftrightarrow \left[\begin{array}{l} x-\frac{\pi}{6}=\frac{\pi}{4}+k2 \\ x-\frac{\pi}{6}=\frac{3\pi}{4}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{5\pi}{12}+k2\pi \\ x=\frac{11\pi}{12}+k2\pi \end{array}\right. ,k\in\mathbb{Z}$$

h. $2\cos 2x - 3\cos x + 1 = 0 \Leftrightarrow 4\cos^2 x - 3\cos x - 1 = 0$

$$\Leftrightarrow \left[\begin{array}{l} \cos x = 1 \\ \cos x = -\dfrac{1}{4} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = k2\pi \\ x = \pm\arccos(-\dfrac{1}{4}) + k2\pi \end{array}\right., k \in \mathbb{Z}$$

i. $2\sin^2 x + 3\sin x\cos x - 5\cos^2 x = 0 \Leftrightarrow 2\tan^2 x + 3\tan x - 5 = 0$

$$\Leftrightarrow \left[\begin{array}{l} \tan x = 1 \\ \tan x = -\dfrac{5}{2} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \dfrac{\pi}{4} + k\pi \\ x = \arctan(-\dfrac{5}{2}) + k\pi \end{array}\right., k \in \mathbb{Z}$$

Bài 3. Giải các phương trình:

a. $3\sin x + \sin 2x = 0$

b. $2sinx - 2\cos x = \sqrt{2}$

c. $\sin x + \sin 3x + \sin 5x = 0$

d. $\sin x + \sin 3x + \sin 5x = \cos x + \cos 3x + \cos 5x$

e. $2\sin^2 x - 5\sin x\cos x - 4\cos^2 x = 2$

f. $2\cos^2 2x + 3\sin^2 x = 2$

g. $\sin^2 2x + \cos^2 3x = 1$

h. $\tan x.\tan 5x = 1$

i. $5\cos 2x - 12\sin 2x = -13$

j. $2\sin x - 5\cos x = 4$

k. $2\cos x + 3\sin x = 2$

Bài 4. Giải các phương trình:

a. $\tan x + \cot x = 2$

b. $(3 + \cot x)^2 = 5(3 + \cot x)$

c. $3(\sin 3x - \cos x) = 4(\cos 3x - \sin x)$

d. $4\sin^2 x + 3\sqrt{3}\sin 2x - 2\cos^2 x = 4$

e. $\sin^2 x + \sin^2 2x + \sin^2 3x + \sin^2 4x = 2$

f. $4\sin^4 x + 12\cos^2 x = 7$

Bài 5. Giaûi caùc phöông trình sau :

a) $\sqrt{2}\cot(5x - \dfrac{\pi}{8}) = 0$

b) $2\cos^2 x + \sqrt{3}\cos x = 0$

c) $\sqrt{3}\sin 3x - \cos 3x = 2$

d) $\sin^2 x + \sin 2x + 2\cos^2 x = 2$

Baøi giaûi :

a) $\sqrt{2}\cot(5x - \dfrac{\pi}{8}) = 0$

$\Leftrightarrow\ 5x-\dfrac{\pi}{8}=\dfrac{\pi}{2}+k\pi \quad \Leftrightarrow \quad x=\pi+\dfrac{k\pi}{5}$

b) $2\cos^2 x+\sqrt{3}\cos x=0$

$\Leftrightarrow \left[\begin{array}{l} \cos x=0 \\ \cos x=-\dfrac{\sqrt{3}}{2} \end{array}\right. \quad \Leftrightarrow \left[\begin{array}{l} x=\dfrac{\pi}{2}+k\pi \\ x=\pm\dfrac{5\pi}{6}+k2\pi \end{array}\right.$

c) $\sqrt{3}\sin 3x-\cos 3x=2$

$\dfrac{\sqrt{3}}{2}\sin 3x-\dfrac{1}{2}\cos 3x=1 \Leftrightarrow \text{Sin}\ \ (3x-\dfrac{\pi}{6})=\ 1 \Leftrightarrow 3x-\dfrac{\pi}{6}=\dfrac{\pi}{2}+k2\pi \Leftrightarrow x=\dfrac{2\pi}{9}+\dfrac{k2\pi}{3}$

d) $\sin^2 x+\sin 2x+2\cos^2 x=2$

$\Leftrightarrow$ **sinx (2 cosx – sinx) = 0** $\quad \Leftrightarrow \left[\begin{array}{l} \sin x=0 \\ \tan x=2 \end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l} x=k\pi \\ x=\arctan 2+k\pi \end{array}\right.$

Bài 6. giai phöông trìnhlöông giac :

a) $\sqrt{3}\tan(3x+\dfrac{3\pi}{5})=0 \quad \Leftrightarrow\ 3x+\dfrac{3\pi}{5}=k\pi \ \Leftrightarrow\ x=-\dfrac{\pi}{5}+\dfrac{k\pi}{3}$

b) $2\sin^2 x-\sin x-1=0 \Leftrightarrow \left[\begin{array}{l} \sin x=1 \\ \sin x=-\dfrac{1}{2} \end{array}\right. \quad \Leftrightarrow \left[\begin{array}{l} x=\dfrac{\pi}{2}+k2\pi \\ x=-\dfrac{\pi}{6}+k2\pi \\ x=\dfrac{7\pi}{6}+k2\pi \end{array}\right.$

c) $\sin 5x+\cos 5x=-\sqrt{2} \quad \dfrac{1}{\sqrt{2}}\sin 5x+\dfrac{1}{\sqrt{2}}\cos 5x=-1 \quad \Leftrightarrow \text{Sin}\ \ (5x+\dfrac{\pi}{4})=\ -1$

$\Leftrightarrow 5x+\dfrac{\pi}{4}=-\dfrac{\pi}{2}+k2\pi \Leftrightarrow\ x=-\dfrac{3\pi}{20}+\dfrac{k2\pi}{5}$

d) $3\sin^2 x+\sin 2x+\cos^2 x=3 \Leftrightarrow \left[\begin{array}{l} \cos x=0 \\ \tan x=1 \end{array}\right. \quad \Leftrightarrow \left[\begin{array}{l} x=\dfrac{\pi}{2}+k\pi \\ x=\dfrac{\pi}{4}+k\pi \end{array}\right.$

Bài 7. Giải các phương trình sau:

a. $2\sin x - 1 = 0$ b. $2\cos x - \sqrt{3} = 0$ c. $\cos 2x + 3\sin x - 2 = 0$ d. $\sqrt{3}\sin x - \cos x = \sqrt{2}$

$$a)\ \sin x = \sin\frac{\pi}{6} \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{6} + k2\pi \\ x = \frac{5\pi}{6} + k2\pi \end{array}\right.$$

$$b)\ \cos x = \cos\frac{\pi}{6} \Leftrightarrow x = \pm\frac{\pi}{6} + k2\pi$$

$$c)\ -2\sin^2 x + 3\sin x - 1 = 0$$

$$\left[\begin{array}{l} \sin x = 1 \\ \sin x = \frac{1}{2} \end{array}\right.$$

$$\left[\begin{array}{l} x = \frac{\pi}{2} + k2\pi \\ x = \frac{\pi}{6} + l2\pi \\ x = \frac{5\pi}{6} + l2\pi \end{array}\right.$$

$$d)\ \frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\left[\begin{array}{l} x = \frac{5\pi}{12} + k2\pi \\ x = \frac{11\pi}{12} + k2\pi \end{array}\right.$$

Bài 8. Giải các phương trình sau:

a. $2\sin x - \sqrt{3} = 0$ b. $2\cos x - 1 = 0$ c. $\cos 2x + 3\sin x - 2 = 0$ d. $\sqrt{3}\sin x + \cos x = \sqrt{2}$

$$a)\ \sin x = \sin\frac{\pi}{3} \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{3} + k2\pi \\ x = \frac{2\pi}{3} + k2\pi \end{array}\right.$$

$$b)\ \cos x = \cos\frac{\pi}{3} \Leftrightarrow x = \pm\frac{\pi}{3} + k2\pi$$

$$c)\ -2\sin^2 x + 3\sin x - 1 = 0$$

$$\left[\begin{array}{l} \sin x = 1 \\ \sin x = \frac{1}{2} \end{array}\right.$$

$$\left[\begin{array}{l} x = \frac{\pi}{2} + k2\pi \\ x = \frac{\pi}{6} + k2\pi \\ x = \frac{5\pi}{6} + k2\pi \end{array}\right.$$

$$d)\ \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = \frac{\sqrt{2}}{2}$$

$$\left[\begin{array}{l} x = \frac{\pi}{12} + k2\pi \\ x = \frac{7\pi}{12} + k2\pi \end{array}\right.$$

Bài 9. Giải các phương trình sau:

a. $2\sin x - 1 = 0$ b. $2\cos x - \sqrt{2} = 0$ c. 2 cos2x -3cosx +1 =0 d. $\sqrt{3}\sin x - \cos x = \sqrt{2}$

a) $\sin x = \sin\frac{\pi}{6} \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{6} + k2\pi \\ x = \frac{5\pi}{6} + k2\pi \end{array}\right.$

b) $\cos x = \cos\frac{\pi}{4} \Leftrightarrow x = \pm\frac{\pi}{4} + k2\pi$

c) $4\cos^2 x - 3\cos x - 1 = 0$

$\left[\begin{array}{l} \cos x = 1 \\ \cos x = -\frac{1}{4} \end{array}\right.$

$\left[\begin{array}{l} x = k2\pi \\ x = \pm\arccos\left(-\frac{1}{4}\right) + k2\pi \end{array}\right.$

d) $\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x = \frac{\sqrt{2}}{2}$

$\left[\begin{array}{l} x = \frac{5\pi}{12} + k2\pi \\ x = \frac{11\pi}{12} + k2\pi \end{array}\right.$

Bài 10. Giải Phương trình

a. $\sqrt{3}\sin x - \cos x = \sqrt{2}$ b. $\cos 2x + 3\sin x - 2 = 0$

c. $\cos^2$x + sinx +1=0

a/ $\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x = \frac{\sqrt{2}}{2}$ $\sin\left(x - \frac{\pi}{6}\right) = \sin\frac{\pi}{4} \Leftrightarrow \left[\begin{array}{l} x = \frac{5\pi}{12} + k2\pi \\ x = \frac{11\pi}{12} + k2\pi \end{array}\right.$

b $-2\sin^2 x + 3\sin x - 1 = 0$ $\left[\begin{array}{l} \sin x = 1 \\ \sin x = \frac{1}{2} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{2} + k2\pi \\ x = \frac{\pi}{6} + l2\pi \\ x = \frac{5\pi}{6} + l2\pi \end{array}\right.$

c. $\left[\begin{array}{l} x = \frac{\pi}{4} + k\pi \\ x = \frac{\pi}{6} + k\pi \end{array}\right.$

Bài 11. Giải các pt.

a. $\cos 2x + 3\sin x - 2 = 0$ b.$\sin^2$x +3sinx cosx -5 $\cos^2$x= 0

c.2 $\cos^2$x -3cosx +1 =0

Đáp án

a $-2\sin^2 x+3\sin x-1=0 \Leftrightarrow \left[\begin{array}{l}\sin x=1\\ \sin x=\frac{1}{2}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{2}+k2\pi\\ x=\frac{\pi}{6}+l2\pi\\ x=\frac{5\pi}{6}+l2\pi\end{array}\right.$

b $t=\sin x-\cos x,\ -\sqrt{2}\le t\le\sqrt{2}$ => $\sin x.\cos x=\frac{1-t^2}{2}$

PT $\Leftrightarrow -t^2+12t-11=0$ => $\left[\begin{array}{l}t=1\\ t=11(loai)\end{array}\right.$ => $\left[\begin{array}{l}x=\frac{\pi}{2}+k2\pi\\ x=\pi+k2\pi\end{array}\right.$

c. $\left[\begin{array}{l}x=k2\pi\\ x=\pm\frac{\pi}{3}+k2\pi\end{array}\right.$

Bài 12. a. Giải các Phương trình sau: $2\cos\left(x+\frac{\pi}{3}\right)+1=0$

b.$\sin^2x+3\sin x\cos x-5\cos^2x=0$

a/ $2\cos\left(x+\frac{\pi}{3}\right)+1=0 \Leftrightarrow \cos\left(x+\frac{\pi}{3}\right)=-\frac{1}{2}=\cos\frac{2\pi}{3} \Leftrightarrow \left\{\begin{array}{l}x=\frac{\pi}{3}+k2\pi\\ x=-\pi+k2\pi\end{array}\right.$

b/ $t=\sin x-\cos x,\ -\sqrt{2}\le t\le\sqrt{2}$

$\sin x.\cos x=\frac{1-t^2}{2}$

PT $\Leftrightarrow -t^2+12t-11=0$

$\left[\begin{array}{l}t=1\\ t=11(loai)\end{array}\right.$ => $\left[\begin{array}{l}x=\frac{\pi}{2}+k2\pi\\ x=\pi+k2\pi\end{array}\right.$

Bài 13. Giải các phương trình sau

a. $2\sin^2 x-3\sin x+1=0$ b. $3\sin x+\sin 2x=0$ c. $2\sin x-2\cos x=\sqrt{2}$

Đs a. $\left[\begin{array}{l} x=\dfrac{\pi}{2}+k2\pi \\ x=\dfrac{\pi}{6}+k2\pi \end{array}\right.$ b. x=k360^0 c. $\left[\begin{array}{l} x=\dfrac{5\pi}{24}+k\pi \\ x=\dfrac{13\pi}{24}+k\pi \end{array}\right.$

Bài 13. Giải Phương trình

a. $\tan(x+20^0)=\dfrac{1}{2}$ b. sinx + sin2x = cosx + cos3x

c.$4\sin^2x-5\sin x\cos x-6\cos^2x=0$

ĐS.a. x=10^0 +k180^0

b. $\left[\begin{array}{l} x=\pi+k2\pi \\ x=\dfrac{\pi}{6}+k\dfrac{2\pi}{3} \end{array}\right.$ c. $\left[\begin{array}{l} x=\arctan 2+k\pi \\ x=\arctan(-\dfrac{1}{2})+k\pi \end{array}\right.$

Bài 14. Giải Phương trình

a. $\sqrt{3}\sin x-\cos x=\sqrt{2}$ b. $\cos 2x+3\sin x-2=0$

1a) $\dfrac{\sqrt{3}}{2}\sin x-\dfrac{1}{2}\cos x=\dfrac{\sqrt{2}}{2}$ $\Leftrightarrow \sin\left(x-\dfrac{\pi}{6}\right)=\sin\dfrac{\pi}{4}$ $\Leftrightarrow \left[\begin{array}{l} x=\dfrac{5\pi}{12}+k2\pi \\ x=\dfrac{11\pi}{12}+k2\pi \end{array}\right.$

1b) $-2\sin^2 x+3\sin x-1=0$

$$\left[\begin{array}{l} \sin x=1 \\ \sin x=\dfrac{1}{2} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\dfrac{\pi}{2}+k2\pi \\ x=\dfrac{\pi}{6}+l2\pi \\ x=\dfrac{5\pi}{6}+l2\pi \end{array}\right.$$

Bài 15. Giải pt: a. $4\tan^2 x-7\tan x+3=0$ b.$\sin(2x+\dfrac{\pi}{3})=-\dfrac{\sqrt{2}}{2}$

<u>Đáp án :</u> **a.** $\sin(3x-\dfrac{\pi}{6})\neq 0(0.25)\Leftrightarrow 3x-\dfrac{\pi}{6}\neq k\pi(0.25), x\neq\dfrac{\pi}{18}+\dfrac{k\pi}{3}(0.5)$

b. $\left[\begin{array}{l} 2x+\dfrac{\pi}{3}=-\dfrac{\pi}{4}+k2\pi \\ 2x+\dfrac{\pi}{3}=\dfrac{5\pi}{4}+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=-\dfrac{7\pi}{24}+k\pi \\ x=\dfrac{11\pi}{24}+k\pi \end{array}\right. (0.25*4)$

Bài 16. Giải pt: a. $2\cot^2 x - 5\cot x + 3 = 0$ b. $\cos(2x + \frac{\pi}{3}) = -\frac{\sqrt{2}}{2}$ c. $2\cos^2 2x + 3\sin^2 = 2$

Đáp án : **a.** $\cos(3x - \frac{\pi}{6}) \neq 0(0.25) \Leftrightarrow 3x - \frac{\pi}{6} \neq \frac{\pi}{2} + k\pi(0.25), x \neq \frac{2\pi}{18} + \frac{k\pi}{3}(0.5)$

$$\begin{cases}\cos x = 1 \\ \cot x = \frac{3}{2}\end{cases} \begin{cases} x = \frac{\pi}{4} + k\pi \\ x = arc\cot\frac{3}{2} + k\pi\end{cases}$$

b. $\left[\begin{array}{l} 2x + \frac{\pi}{3} = -\frac{\pi}{4} + k2\pi \\ 2x + \frac{\pi}{3} = \frac{\pi}{4} + k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = -\frac{7\pi}{24} + k\pi \\ x = -\frac{\pi}{24} + k\pi \end{array}\right. (0.25*4)$

c.

$$4\cos^2 2x - 3\cos 2x - 1 = 0 \Leftrightarrow \begin{cases}\cos 2x = 1 \\ \cos 2x = -\frac{1}{4}\end{cases}$$

$$\Leftrightarrow \begin{cases} 2x = 2k\pi \\ 2x = \pm\arccos(-\frac{1}{4}) + 2k\pi \end{cases} \Leftrightarrow \begin{cases} x = k\pi \\ x = \pm\frac{1}{2}\arccos(-\frac{1}{4}) + k\pi \end{cases} k \in Z$$

$5\sin x - \sin^5 x = 0$ h. $\cos 7x - \sin 5x = \sqrt{3}(\cos 5x - \sin 7x)$

Phương trình asinx + bcosx = c

Bài 1. $\cos 7x - \sqrt{3}\sin 7x = -\sqrt{2} \Leftrightarrow \left[\begin{array}{l} x = \frac{5\pi}{84} + k\frac{2\pi}{7} \\ x = \frac{11\pi}{84} + k\frac{2\pi}{7} \end{array}\right.$

Bài 2. $3(\sin 5x - \cos x) = 4(\sin x + \cos 5x) \Leftrightarrow 3\sin 5x - 4\cos 5x = 4\sin x + 3\cos x$

$$\Leftrightarrow \frac{3}{5}\sin 5x - \frac{4}{5}\cos 5x = \frac{4}{5}\sin x + \frac{3}{5}\cos x$$

$$\Leftrightarrow \sin 5x\cos\alpha - \cos 5x\sin\alpha = \sin x\sin\alpha + \cos x\cos\alpha\ ,\ (\frac{3}{5} = \cos\alpha, \frac{4}{5} = \sin\alpha)$$

$$\Leftrightarrow \sin(5x-\alpha)=\cos(x-\alpha) \Leftrightarrow \sin(5x-\alpha)=\sin(\frac{\pi}{2}-x+\alpha)$$

$$\Leftrightarrow \left[\begin{array}{l} 5x-\alpha=\frac{\pi}{2}-x+\alpha+k2\pi \\ 5x-\alpha=\pi-\frac{\pi}{2}+x-\alpha+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{12}+\frac{\alpha}{3}+k\frac{\pi}{3} \\ x=\frac{\pi}{8}+k\frac{\pi}{2} \end{array}\right.$$

Bài 3. $3\sin 3x-\sqrt{3}\cos 9x=1+4\sin^3 3x \Leftrightarrow (3\sin 3x-4\sin^3 3x)-\sqrt{3}\cos 9x=1$

$$\Leftrightarrow \sin 9x-\sqrt{3}\cos 9x=1 \Leftrightarrow \sin(9x-\frac{\pi}{3})=\sin\frac{\pi}{6} \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{18}+k\frac{2\pi}{9} \\ x=\frac{7\pi}{54}+k\frac{2\pi}{9} \end{array}\right.$$

Bài 4. $\tan x-\sin 2x-\cos 2x+2(2\cos x-\frac{1}{\cos x})=0 \quad (1)$

Điều kiện: $\cos x\neq 0 \Leftrightarrow x\neq\frac{\pi}{2}+k\pi$

$$(1)\Leftrightarrow \frac{\sin x}{\cos x}-\sin 2x-\cos 2x+4\cos x-\frac{2}{\cos x}=0$$

$$\Leftrightarrow \sin x-2\sin x\cos^2 x-\cos 2x\cos x+2(2\cos^2 x-1)=0$$

$$\Leftrightarrow \sin x(1-2\cos^2 x)-\cos 2x\cos x+2\cos 2x=0$$

$$\Leftrightarrow -\sin x\cos 2x-\cos 2x\cos x+2\cos 2x=0$$

$$\Leftrightarrow \cos 2x(\sin x+\cos x-2)=0 \Leftrightarrow \left[\begin{array}{l} \cos 2x=0 \\ \sin x+\cos x=2 (vn) \end{array}\right. \Leftrightarrow x=\frac{\pi}{4}+k\frac{\pi}{2}$$

Bài 5. $8\sin x=\frac{\sqrt{3}}{\cos x}+\frac{1}{\sin x} \quad (*)$

Điều kiện: $\sin 2x\neq 0 \Leftrightarrow x\neq k\frac{\pi}{2}$

$$(*)\Leftrightarrow 8\sin^2 x\cos x=\sqrt{3}\sin x+\cos x \Leftrightarrow 4(1-\cos 2x)\cos x=\sqrt{3}\sin x+\cos x$$

$$\Leftrightarrow -4\cos 2x\cos x=\sqrt{3}\sin x-3\cos x \Leftrightarrow -2(\cos 3x+\cos x)=\sqrt{3}\sin x-3\cos x$$

$$\Leftrightarrow \cos 3x = \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x \Leftrightarrow \cos 3x = \cos(x + \frac{\pi}{3}) \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{6} + k\pi \\ x = -\frac{\pi}{12} + k\frac{\pi}{2} \end{array}\right.$$

C2 $(*) \Leftrightarrow 8\sin^2 x\cos x = \sqrt{3}\sin x + \cos x \Leftrightarrow 8(1 - \cos^2 x)\cos x = \sqrt{3}\sin x + \cos x$

$$\Leftrightarrow 8\cos x - 8\cos^3 x = \sqrt{3}\sin x - 3\cos x \Leftrightarrow 6\cos x - 8\cos^3 x = \sqrt{3}\sin x - \cos x$$

$$\Leftrightarrow 4\cos^3 x - 3\cos x = \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x \Leftrightarrow \cos 3x = \cos(x + \frac{\pi}{3})$$

$$\Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{6} + k\pi \\ x = -\frac{\pi}{12} + k\frac{\pi}{2} \end{array}\right.$$

Bài 6. $9\sin x + 6\cos x - 3\sin 2x + \cos 2x = 8$

$$\Leftrightarrow 6\sin x\cos x - 6\cos x + 2\sin^2 x - 9\sin x + 7 = 0$$

$$\Leftrightarrow 6\cos x(\sin x - 1) + (\sin x - 1)(2\sin x - 7) = 0$$

$$\Leftrightarrow (\sin x - 1)(6\cos x + 2\sin x - 7) = 0$$

$$\Leftrightarrow \left[\begin{array}{l} \sin x = 1 \\ 6\cos x + 2\sin x = 7 \end{array}\right. \Leftrightarrow x = \frac{\pi}{2} + k2\pi$$

Bài 7. $\sin 2x + 2\cos 2x = 1 + \sin x - 4\cos x$

$$\Leftrightarrow 2\sin x\cos x + 2(2\cos^2 x - 1) - 1 - \sin x + 4\cos x = 0$$

$$\Leftrightarrow \sin x(2\cos x - 1) + 4\cos^2 x + 4\cos x - 3 = 0$$

$$\Leftrightarrow \sin x(2\cos x - 1) + (2\cos x - 1)(2\cos x + 3) = 0$$

$$\Leftrightarrow (2\cos x - 1)(2\sin x + 2\cos x + 3) = 0$$

$$\Leftrightarrow \left[\begin{array}{l} \cos x = \frac{1}{2} \\ 2\sin x + 2\cos x = -3, (vn) \end{array}\right. \Leftrightarrow x = \pm\frac{\pi}{3} + k2\pi$$

Bài 8. $2\sin 2x - \cos 2x = 7\sin x + 2\cos x - 4$

$$\Leftrightarrow 4\sin x\cos x-(1-2\sin^2 x)-7\sin x-2\cos x+4=0$$

$$\Leftrightarrow 2\cos x(2\sin x-1)+(2\sin^2 x-7\sin x+3)=0$$

$$\Leftrightarrow 2\cos x(2\sin x-1)+(2\sin x-1)(\sin x-3)=0$$

$$\Leftrightarrow (2\sin x-1)(2\cos x+\sin x-3)=0$$

$$\Leftrightarrow \left[\begin{array}{l} 2\sin x-1=0 \\ 2\cos x+\sin x=3,(vn) \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\dfrac{\pi}{6}+k2\pi \\ x=\dfrac{5\pi}{6}+k2\pi \end{array}\right.$$

Bài 9. $\sin 2x-\cos 2x=3\sin x+\cos x-2$

$$\Leftrightarrow 2\sin x\cos x-(1-2\sin^2 x)-3\sin x-\cos x+2=0$$

$$\Leftrightarrow (2\sin x\cos x-\cos x)+(2\sin^2 x-3\sin x+1)=0$$

$$\Leftrightarrow \cos x(2\sin x-1)+(2\sin x-1)(\sin x-1)=0$$

$$\Leftrightarrow (2\sin x-1)(\cos x+\sin x-1)=0 \Leftrightarrow \left[\begin{array}{l} 2\sin x=1 \\ \cos x+\sin x=1 \end{array}\right.$$

$$+2\sin x=1 \Leftrightarrow \left[\begin{array}{l} x=\dfrac{\pi}{6}+k2\pi \\ x=\dfrac{5\pi}{6}+k2\pi \end{array}\right.$$

$$+\cos x+\sin x=1 \Leftrightarrow \cos(x-\frac{\pi}{4})=\frac{\sqrt{2}}{2} \Leftrightarrow \left[\begin{array}{l} x=k2\pi \\ x=\dfrac{\pi}{2}+k2\pi \end{array}\right.$$

Bài 10. $(\sin 2x+\sqrt{3}\cos 2x)^2-5=\cos(2x-\frac{\pi}{6})$

Ta có: $\sin 2x+\sqrt{3}\cos 2x=2(\frac{1}{2}\sin 2x+\frac{\sqrt{3}}{2}\cos 2x)=2\cos(2x-\frac{\pi}{6})$

Đặt: $t=\sin 2x+\sqrt{3}\cos 2x,-2\le t\le 2$

Phương trình trở thành: $t^2-5=\frac{t}{2} \Leftrightarrow 2t^2-t-10=0 \Leftrightarrow \left[\begin{array}{l} t=-2 \\ t=\frac{5}{2} \end{array}\right.$

$+t=\frac{5}{2}$: loại

$+t=-2: 2\cos(2x-\frac{\pi}{6})=-2 \Leftrightarrow x=\frac{7\pi}{12}+k\pi$

Bài 11. $2\cos^3 x+\cos 2x+\sin x=0 \Leftrightarrow 2\cos^3 x+2\cos^2 x-1+\sin x=0$

$\Leftrightarrow 2\cos^2 x(\cos x+1)-(1-\sin x)=0 \Leftrightarrow 2(1-\sin^2 x)(\cos x+1)-(1-\sin x)=0$

$\Leftrightarrow 2(1-\sin x)(1+\sin x)(\cos x+1)-(1-\sin x)=0$

$\Leftrightarrow (1-\sin x)[2(1+\sin x)(\cos x+1)-1]=0$

$\Leftrightarrow (1-\sin x)[1+2\sin x\cos x+2(\sin x+\cos x)]=0$

$\Leftrightarrow \left[\begin{array}{c} \sin x=1 \\ 1+2\sin x\cos x+2(\sin x+\cos x)=0 \end{array}\right.$

$+\sin x=1 \Leftrightarrow x=\frac{\pi}{2}+k2\pi$

$+1+2\sin x\cos x+2(\sin x+\cos x)=0 \Leftrightarrow (\sin x+\cos x)^2+2(\sin x+\cos x)=0$

$\Leftrightarrow (\sin x+\cos x)(\sin x+\cos x+2)=0 \Leftrightarrow \sin x+\cos x=0$

$\Leftrightarrow \tan x=-1 \Leftrightarrow x=-\frac{\pi}{4}+k\pi$

Bài 12. $1+\cot 2x=\frac{1-\cos 2x}{\sin^2 2x}$ (*) Điều kiện: $\sin 2x\neq 0 \Leftrightarrow x\neq k\frac{\pi}{2}$

$(*) \Leftrightarrow 1+\cot 2x=\frac{1-\cos 2x}{1-\cos^2 2x} \Leftrightarrow 1+\cot 2x=\frac{1}{1+\cos 2x} \Leftrightarrow 1+\frac{\cos 2x}{\sin 2x}=\frac{1}{1+\cos 2x}$

$\Leftrightarrow \sin 2x(1+\cos 2x)+\cos 2x(1+\cos 2x)=\sin 2x$

$\Leftrightarrow \sin 2x\cos 2x+\cos 2x(1+\cos 2x)=0 \Leftrightarrow \cos 2x(\sin 2x+\cos 2x+1)=0$

$\Leftrightarrow \left[\begin{array}{c} \cos 2x=0 \\ \sin 2x+\cos 2x=-1 \end{array}\right.$

$+\cos 2x=0 \Leftrightarrow x=\frac{\pi}{4}+k\frac{\pi}{2}$

$+\sin 2x+\cos 2x=-1 \Leftrightarrow \sin(2x+\frac{\pi}{4})=\sin(-\frac{\pi}{4}) \Leftrightarrow \left[\begin{array}{l} x=-\frac{\pi}{4}+k\pi \\ x=\frac{\pi}{2}+k\pi \end{array}\right.$

Vậy,phương trình có nghiệm: $x=\frac{\pi}{4}+k\frac{\pi}{2}$

Bài 13. $4(\sin^4 x+\cos^4 x)+\sqrt{3}\sin 4x=2$

$\Leftrightarrow 4[(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x]+\sqrt{3}\sin 4x=2$

$\Leftrightarrow 4(1-\frac{1}{2}\sin^2 2x)+\sqrt{3}\sin 4x=2 \Leftrightarrow \cos 4x+\sqrt{3}\sin 4x=-2$

$\Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{4}+k\frac{\pi}{2} \\ x=-\frac{\pi}{12}+k\frac{\pi}{2} \end{array}\right.$

Bài 14. $1+\sin^3 2x+\cos^3 2x=\frac{1}{2}\sin 4x$

$\Leftrightarrow 2-\sin 4x+2(\sin 2x+\cos 2x)(1-\sin 2x\cos 2x)=0$

$\Leftrightarrow (2-\sin 4x)+(\sin 2x+\cos 2x)(2-\sin 4x)=0$

$\Leftrightarrow (2-\sin 4x)(\sin 2x+\cos 2x+1)=0 \Leftrightarrow \sin 2x+\cos 2x=-1$

$\Leftrightarrow \sin(2x+\frac{\pi}{4})=-\frac{\sqrt{2}}{2} \Leftrightarrow \left[\begin{array}{l} x=-\frac{\pi}{4}+k\pi \\ x=\frac{\pi}{2}+k\pi \end{array}\right.$

Bài 15. $\tan x-3\cot x=4(\sin x+\sqrt{3}\cos x)$ (*) Điều kiện: $\sin 2x\neq 0 \Leftrightarrow x\neq k\frac{\pi}{2}$

$(*) \Leftrightarrow \frac{\sin x}{\cos x}-3\frac{\cos x}{\sin x}=4(\sin x+\sqrt{3}\cos x)$

$\Leftrightarrow \sin^2 x-3\cos^2 x-4\sin x\cos x(\sin x+\sqrt{3}\cos x)=0$

$$\Leftrightarrow (\sin x-\sqrt{3}\cos x)(\sin x+\sqrt{3}\cos x)-4\sin x\cos x(\sin x+\sqrt{3}\cos x)=0$$

$$\Leftrightarrow (\sin x+\sqrt{3}\cos x)(\sin x-\sqrt{3}\cos x-4\sin x\cos x)=0$$

$$\Leftrightarrow \left[\begin{array}{l}\sin x+\sqrt{3}\cos x=0\\ \sin x-\sqrt{3}\cos x-4\sin x\cos x=0\end{array}\right.$$

$+\sin x+\sqrt{3}\cos x=0\Leftrightarrow \tan x=-\sqrt{3}\Leftrightarrow x=-\dfrac{\pi}{3}+k\pi$

$+\sin x-\sqrt{3}\cos x-4\sin x\cos x=0\Leftrightarrow 2\sin 2x=\sin x-\sqrt{3}\cos x$

$$\Leftrightarrow \sin 2x=\frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\cos x\Leftrightarrow \sin 2x=\sin(x-\frac{\pi}{3})\Leftrightarrow \left[\begin{array}{l}x=-\dfrac{\pi}{3}+k2\pi\\ x=\dfrac{4\pi}{9}+k\dfrac{2\pi}{3}\end{array}\right.$$

Vậy,phương trình có nghiệm là: $x=-\dfrac{\pi}{3}+k\pi;\ x=\dfrac{4\pi}{9}+k\dfrac{2\pi}{3}$

Bài 16. $\sin^3 x+\cos^3 x=\sin x-\cos x\Leftrightarrow \sin x(\sin^2 x-1)+\cos^3 x+\cos x=0$

$$\Leftrightarrow -\sin x\cos^2 x+\cos^3 x+\cos x=0\Leftrightarrow \cos x(-\sin x\cos x+\cos^2 x+1)=0$$

$$\Leftrightarrow \left[\begin{array}{l}\cos x=0\\ -\sin x\cos x+\cos^2 x=-1\end{array}\right.$$

$+\cos x=0\Leftrightarrow x=\dfrac{\pi}{2}+k\pi$

$+-\sin x\cos x+\cos^2 x=-1\Leftrightarrow -\dfrac{1}{2}\sin 2x+\dfrac{1+\cos 2x}{2}=-1\Leftrightarrow \sin 2x-\cos 2x=3,(vn)$

Vậy,phương trình có nghiệm là: $x=\dfrac{\pi}{2}+k\pi,k\in\mathbb{Z}$

Bài 17. $\cos^4 x+\sin^4(x+\dfrac{\pi}{4})=\dfrac{1}{4}\ \Leftrightarrow \dfrac{1}{4}(1+\cos 2x)^2+\dfrac{1}{4}[1-\cos(2x+\dfrac{\pi}{2})]^2=\dfrac{1}{4}$

$$\Leftrightarrow (1+\cos 2x)^2+(1+\sin 2x)^2=1\ \Leftrightarrow \sin 2x+\cos 2x=-1$$

$$\Leftrightarrow \cos(2x-\frac{\pi}{4})=\cos\frac{3\pi}{4} \Leftrightarrow \left[\begin{array}{l} x=\frac{\pi}{2}+k2\pi \\ x=-\frac{\pi}{4}+k\pi \end{array}\right.$$

Bài 18. $4\sin^3 x\cos 3x+4\cos^3 x\sin 3x+3\sqrt{3}\cos 4x=3$

$$\Leftrightarrow 4\sin^3 x(4\cos^3 x-3\cos x)+4\cos^3 x(3\sin x-4\sin^3 x)+3\sqrt{3}\cos 4x=3$$

$$\Leftrightarrow -12\sin^3 x\cos x+12\cos^3 x\sin x+3\sqrt{3}\cos 4x=3$$

$$\Leftrightarrow 4\sin x\cos x(\cos^2 x-\sin^2 x)+\sqrt{3}\cos 4x=1$$

$$\Leftrightarrow 2\sin 2x\cos 2x+\sqrt{3}\cos 4x=1 \Leftrightarrow \sin 4x+\sqrt{3}\cos 4x=1$$

$$\Leftrightarrow \frac{1}{2}\sin 4x+\frac{\sqrt{3}}{2}\cos 4x=\frac{1}{2} \Leftrightarrow \sin(4x+\frac{\pi}{3})=\sin\frac{\pi}{6} \Leftrightarrow \left[\begin{array}{l} x=-\frac{\pi}{24}+k\frac{\pi}{2} \\ x=\frac{\pi}{8}+k\frac{\pi}{2} \end{array}\right.,k\in\mathbb{Z}$$

Bài 19. Cho phương trình: $2\sin^2 x-\sin x\cos x-\cos^2 x=m$ (*)

a.Tìm m sao cho phương trình có nghiệm.

b.Giải phương trình khi m = -1.

Giải.

$$(*)\Leftrightarrow (1-\cos 2x)-\frac{1}{2}\sin 2x-\frac{1}{2}(1+\cos 2x)=m \Leftrightarrow \sin 2x+3\cos 2x=-2m+1$$

a. (*)có nghiệm khi: $c^2\le a^2+b^2 \Leftrightarrow (1-2m)^2\le 1+9 \Leftrightarrow 4m^2-4m-9\le 0$

$$\Leftrightarrow \frac{1-\sqrt{10}}{2}\le m\le\frac{1+\sqrt{10}}{2}$$

b.Khi m = -1 phương trình trở thành:

$$\sin 2x+3\cos 2x=3 \Leftrightarrow \frac{1}{\sqrt{10}}\sin 2x+\frac{3}{\sqrt{10}}\cos 2x=\frac{3}{\sqrt{10}}$$

$$\Leftrightarrow \sin 2x\cos\alpha+\cos 2x\sin\alpha=\sin\alpha, (\frac{1}{\sqrt{10}}=\cos\alpha,\frac{3}{\sqrt{10}}=\sin\alpha)$$

$$\Leftrightarrow \sin(2x+\alpha)=\sin\alpha \Leftrightarrow \left[\begin{array}{l} 2x+\alpha=\alpha+k2\pi \\ 2x+\alpha=\pi-\alpha+k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=k\pi \\ x=\dfrac{\pi}{2}-\alpha+k\pi \end{array}\right.$$

Bài 20. Cho phương trình: $\dfrac{5+4\sin(\frac{3\pi}{2}-x)}{\sin x}=\dfrac{6\tan\alpha}{1+\tan^2\alpha}$ (*)

a.Giải phương trình khi $\alpha=-\dfrac{\pi}{4}$

b.Tìm để phương trình (*) có nghiệm

Giải.

Ta có: $\sin(\dfrac{3\pi}{2}-x)=-\sin(\dfrac{\pi}{2}-x)=-\cos x$

$$\frac{6\tan\alpha}{1+\tan^2\alpha}=6\tan\alpha\cos^2\alpha=3\sin 2\alpha, \cos\alpha\neq 0$$

$$(*)\Leftrightarrow \frac{5-4\cos x}{\sin x}=3\sin 2\alpha \Leftrightarrow 3\sin 2\alpha\sin x+4\cos x=5 \quad (**)$$

a. khi $\alpha=-\dfrac{\pi}{4}$ phương trình trở thành:

$$3\sin x-4\cos x=-5 \Leftrightarrow \frac{3}{5}\sin x-\frac{4}{5}\cos x=-1$$

$$\Leftrightarrow \sin x\cos\alpha-\cos x\sin\alpha=-1, (\frac{3}{5}=\cos\alpha, \frac{4}{5}=\sin\alpha)$$

$$\Leftrightarrow \sin(x-\alpha)=-1 \Leftrightarrow x=\alpha-\frac{\pi}{2}+k2\pi$$

b.Phương trình có nghiệm khi:

$$\begin{cases} \cos\alpha\neq 0 \\ (3\sin 2\alpha)^2+16\geq 25 \end{cases} \Leftrightarrow \begin{cases} \cos\alpha\neq 0 \\ \sin^2 2\alpha\geq 1 \end{cases} \Leftrightarrow \begin{cases} \cos\alpha\neq 0 \\ \sin^2 2\alpha=1 \end{cases} \Leftrightarrow \cos 2\alpha=0 \Leftrightarrow \alpha=\frac{\pi}{4}+k\frac{\pi}{2}$$

Bài 21. Giải các phương trình:

a. $2\sqrt{2}(\sin x+\cos x)\cos x=3+\cos 2x$ b. $(2\cos x-1)(\sin x+\cos x)=1$

c. $2\cos 2x=\sqrt{6}(\cos x-\sin x)$ d. $3\sin x=3-\sqrt{3}\cos x$

e. $2\cos 3x+\sqrt{3}\sin x+\cos x=0$

f. $\cos x+\sqrt{3}\sin x=\sin 2x+\cos x+\sin x$

g. $\cos x+\sqrt{3}\sin x=\dfrac{3}{\cos x+\sqrt{3}\sin x+1}$

h. $\sin x+\cos x=\cos 2x$

i. $4\sin^3 x-1=3\sin x-\sqrt{3}\cos 3x$

j. $3\cos x+4\sin x+\dfrac{6}{3\cos x+4\sin x+1}=6$

k. $\cos 7x\cos 5x-\sqrt{3}\sin 2x=1-\sin 7x\sin 5x$

l. $4(\cos^4 x+\sin^4 x)+\sqrt{3}\sin 4x=2$

m. $\cos^2 x-\sqrt{3}\sin 2x=1+\sin^2 x$

n. $4\sin 2x-3\cos 2x=3(4\sin x-1)$

p. $\dfrac{(2-\sqrt{3})\cos x-2\sin^2(\frac{x}{2}-\frac{\pi}{4})}{2\cos x-1}=1$

q. $\tan x-\sin 2x-\cos 2x=-4\cos x+\dfrac{2}{\cos x}$

Bài 22. Cho phương trình: $\dfrac{m\sin x-2}{m-2\cos x}=\dfrac{m\cos x-2}{m-2\sin x}$ (*)

a.Giải phương trình khi m = 1

b.Tìm để phương trình (*) có nghiệm

Bài 23. Cho phương trình: $\sin x+m\cos x=2$ (*)

a.Giải phương trình khi $m=\sqrt{3}$

b.Tìm để phương trình (*) có nghiệm

Bài 24. Cho phương trình: $\dfrac{2\sin x+\cos x+1}{\sin x-2\cos x+3}=m$ (*)

a.Giải phương trình khi $m=\dfrac{1}{3}$

b.Tìm để phương trình (*) có nghiệm.

PHƯƠNG TRÌNH BẬC HAI ĐỐI VỚI MỘT HÀM SỐ LƯỢNG GIÁC

Bài 1. $5(\sin x+\dfrac{\cos 3x+\sin 3x}{1+2\sin 2x})=3+\cos 2x$ (1)

Điều kiện: $\sin 2x \neq -\frac{1}{2} \Leftrightarrow \left[\begin{array}{l} x \neq -\frac{\pi}{12} + k\pi \\ x \neq \frac{7\pi}{12} + k\pi \end{array}\right., k \in \mathbb{Z}$

Ta có: $5(\sin x + \frac{\cos 3x + \sin 3x}{1 + 2\sin 2x}) = 5\frac{\sin x + 2\sin 2x \sin x + \cos 3x + \sin 3x}{1 + 2\sin 2x}$

$$= 5\frac{\sin x + \cos x - \cos 3x + \cos 3x + \sin 3x}{1 + 2\sin 2x}$$

$$= 5\frac{(\sin 3x + \sin x) + \cos x}{1 + 2\sin 2x} = 5\frac{2\sin 2x \cos x + \cos x}{1 + 2\sin 2x}$$

$$= 5\frac{(2\sin x + 1)\cos x}{1 + 2\sin 2x} = 5\cos x$$

$(1) \Leftrightarrow 5\cos x = \cos 2x + 3 \Leftrightarrow 2\cos^2 x - 5\cos x + 2 = 0$

$\Leftrightarrow \cos x = \frac{1}{2} \Leftrightarrow x = \pm\frac{\pi}{3} + k2\pi$

Bài 2. $\cos^2 3x \cos 2x - \cos^2 x = 0 \Leftrightarrow \frac{1}{2}(1 + \cos 6x)\cos 2x - \frac{1}{2}(1 + \cos 2x) = 0$

$\Leftrightarrow \cos 6x \cos 2x - 1 = 0 \quad (*)$

Cách 1: $(*) \Leftrightarrow (4\cos^3 2x - 3\cos 2x)\cos 2x - 1 = 0 \Leftrightarrow 4\cos^4 2x - 2\cos^2 2x - 1 = 0$

$\Leftrightarrow \cos^2 2x = 1 \Leftrightarrow \sin 2x = 0 \Leftrightarrow x = k\frac{\pi}{2}$

Cách 2: $(*) \Leftrightarrow \frac{1}{2}(\cos 8x + \cos 4x) - 1 = 0 \Leftrightarrow \cos 8x + \cos 4x - 2 = 0$

$\Leftrightarrow 2\cos^2 4x + \cos 4x - 3 = 0 \Leftrightarrow \cos 4x = 1 \Leftrightarrow x = k\frac{\pi}{2}$

Cách 3: $(*) \Leftrightarrow \left[\begin{array}{l} \cos 6x = \cos 2x = 1 \\ \cos 6x = \cos 2x = -1 \end{array}\right.$

Cách 4: $(*) \Leftrightarrow \frac{1}{2}(\cos 8x + \cos 4x) - 1 = 0 \Leftrightarrow \cos 8x + \cos 4x = 2$

$\Leftrightarrow \cos 8x = \cos 4x = 1$

Bài 3. $\cos^4 x + \sin^4 x + \cos(x - \frac{\pi}{4})\sin(3x - \frac{\pi}{4}) - \frac{3}{2} = 0$

$\Leftrightarrow (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x + \frac{1}{2}[\sin(4x - \frac{\pi}{2}) + \sin 2x] - \frac{3}{2} = 0$

$\Leftrightarrow 1 - \frac{1}{2}\sin^2 2x + \frac{1}{2}(-\cos 4x + \sin 2x) - \frac{3}{2} = 0$

$\Leftrightarrow -\frac{1}{2}\sin^2 2x - \frac{1}{2}(1 - 2\sin^2 2x) + \frac{1}{2}\sin 2x - \frac{1}{2} = 0$

$\Leftrightarrow \sin^2 2x + \sin 2x - 2 = 0 \Leftrightarrow \sin 2x = 1 \Leftrightarrow x = \frac{\pi}{4} + k\pi$

Bài 4. $5\sin x - 2 = 3(1 - \sin x)\tan^2 x \qquad (1)$

Điều kiện: $\cos x \neq 0 \Leftrightarrow x \neq \frac{\pi}{2} + k\pi$

$(1) \Leftrightarrow 5\sin x - 2 = 3(1 - \sin x)\frac{\sin^2 x}{\cos^2 x} \Leftrightarrow 5\sin x - 2 = 3(1 - \sin x)\frac{\sin^2 x}{1 - \sin^2 x}$

$\Leftrightarrow 5\sin x - 2 = \frac{3\sin^2 x}{1 + \sin x} \Leftrightarrow 2\sin^2 x + 3\sin x - 2 = 0 \Leftrightarrow \sin x = \frac{1}{2}$

$$\Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{6} + k2\pi \\ x = \frac{5\pi}{6} + k2\pi \end{array}\right.$$

Bài 5. $2\sin 3x - \frac{1}{\sin x} = 2\cos 3x + \frac{1}{\cos x} \qquad (*)$

Điều kiện: $\sin 2x \neq 0 \Leftrightarrow x \neq k\frac{\pi}{2}$

$(*) \Leftrightarrow 2(\sin 3x - \cos 3x) = \frac{1}{\sin x} + \frac{1}{\cos x}$

$\Leftrightarrow 2[3(\sin x + \cos x) - 4(\sin^3 x + \cos^3 x] = \frac{1}{\sin x} + \frac{1}{\cos x}$

$$\Leftrightarrow 2(\sin x+\cos x)[3-4(\sin^2 x-\sin x\cos x+\cos^2 x)]=\frac{\sin x+\cos x}{\sin x\cos x}$$

$$\Leftrightarrow 2(\sin x+\cos x)(-1+4\sin x\cos x)-\frac{\sin x+\cos x}{\sin x\cos x}=0$$

$$\Leftrightarrow (\sin x+\cos x)(-2+8\sin x\cos x-\frac{1}{\sin x\cos x})=0$$

$$\Leftrightarrow (\sin x+\cos x)(4\sin 2x-\frac{2}{\sin 2x}-2)=0$$

$$\Leftrightarrow (\sin x+\cos x)(4\sin^2 2x-2\sin 2x-2)=0$$

$$\Leftrightarrow \left[\begin{array}{c}\sin x+\cos x=0\\ 4\sin^2 2x-2\sin 2x-2=0\end{array}\right. \Leftrightarrow \left[\begin{array}{c}\tan x=-1\\ \sin 2x=1\\ \sin 2x=-1/2\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=\pm\frac{\pi}{4}+k\pi\\ x=-\frac{\pi}{12}+k\pi\\ x=\frac{7\pi}{12}+k\pi\end{array}\right.$$

Bài 6. $\dfrac{\cos x(2\sin x+3\sqrt{2})-2\cos^2 x-1}{1+\sin 2x}=1$ (*)

Điều kiện: $\sin 2x\neq -1\Leftrightarrow x\neq -\frac{\pi}{4}+k\pi$

$$(*)\Leftrightarrow 2\sin x\cos x+3\sqrt{2}\cos x-2\cos^2 x-1=1+\sin 2x$$

$$\Leftrightarrow 2\cos^2 x-3\sqrt{2}\cos x+2=0 \Leftrightarrow \cos x=\frac{\sqrt{2}}{2} \Leftrightarrow x=\pm\frac{\pi}{4}+k\pi$$

Đối chiếu điều kiện phương trình có nghiệm: $x=\frac{\pi}{4}+k\pi, k\in\mathbb{Z}$

Bài 7. $\cos x\cos\frac{x}{2}\cos\frac{3x}{2}-\sin x\sin\frac{x}{2}\sin\frac{3x}{2}=\frac{1}{2}$

$$\Leftrightarrow \frac{1}{2}\cos x(\cos 2x+\cos x)+\frac{1}{2}\sin x(\cos 2x-\cos x)=\frac{1}{2}$$

$$\Leftrightarrow \cos x\cos 2x+\cos^2 x+\sin x\cos 2x-\sin x\cos x=1$$

$$\Leftrightarrow \cos 2x(\sin x+\cos x)+1-\sin^2 x-\sin x\cos x-1=0$$

$$\Leftrightarrow \cos 2x(\sin x+\cos x)-\sin x(\sin x+\cos x)=0$$

$$\Leftrightarrow (\sin x+\cos x)(\cos 2x-\sin x)=0$$

$$\Leftrightarrow (\sin x+\cos x)(-2\sin^2 x-\sin x+1)=0$$

$$\Leftrightarrow \left[\begin{array}{l} \sin x+\cos x=0 \\ 2\sin^2 x+\sin x-1=0 \end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l} \tan x=-1 \\ \sin x=-1 \\ \sin x=1/2 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=-\dfrac{\pi}{4}+k\pi \\ x=-\dfrac{\pi}{2}+k2\pi \\ x=\dfrac{\pi}{6}+k2\pi \vee x=\dfrac{5\pi}{6}+k2\pi \end{array}\right.$$

Bài 8. $4\cos^3 x+3\sqrt{2}\sin 2x=8\cos x \Leftrightarrow 4\cos^3 x+6\sqrt{2}\sin x\cos x-8\cos x=0$

$$\Leftrightarrow 2\cos x(2\cos^2 x+3\sqrt{2}\sin x-4)=0 \Leftrightarrow 2\cos x(2\sin^2 x-3\sqrt{2}\sin x+2)=0$$

$$\Leftrightarrow \left[\begin{array}{l} \cos x=0 \\ \sin x=\dfrac{\sqrt{2}}{2} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=\dfrac{\pi}{2}+k\pi \\ x=\dfrac{\pi}{4}+k2\pi \\ x=\dfrac{3\pi}{4}+k2\pi \end{array}\right.$$

Bài 9. $\cos(2x+\dfrac{\pi}{4})+\cos(2x-\dfrac{\pi}{4})+4\sin x=2+\sqrt{2}(1-\sin x)$

$$\Leftrightarrow 2\cos 2x\cos\frac{\pi}{4}+4\sin x-2-\sqrt{2}+\sqrt{2}\sin x=0$$

$$\Leftrightarrow \sqrt{2}(1-2\sin^2 x)+4\sin x-2-\sqrt{2}+\sqrt{2}\sin x=0$$

$$\Leftrightarrow 2\sqrt{2}\sin^2 x-(4+\sqrt{2})\sin x+2=0$$

$$\Leftrightarrow \sin x = \frac{1}{2} \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{6} + k2\pi \\ x = \frac{5\pi}{6} + k2\pi \end{array}\right.$$

Bài 10. $3\cot^2 x + 2\sqrt{2}\sin^2 x = (2+3\sqrt{2})\cos x$ (1)

Điều kiện: $\sin x \neq 0 \Leftrightarrow x \neq k\pi$

$$(1) \Leftrightarrow 3\frac{\cos^2 x}{\sin^4 x} + 2\sqrt{2} = (2+3\sqrt{2})\frac{\cos x}{\sin^2 x}$$

Đặt: $t = \frac{\cos x}{\sin^2 x}$ phương trình trở thành: $3t^2 - (2+3\sqrt{2})t + 2\sqrt{2} = 0 \Leftrightarrow \left[\begin{array}{l} t = \sqrt{2} \\ t = \frac{2}{3} \end{array}\right.$

$+t = \frac{2}{3} : \frac{\cos x}{\sin^2 x} = \frac{2}{3} \Leftrightarrow 3\cos x = 2(1-\cos^2 x) \Leftrightarrow 2\cos^2 x + 3\cos x - 2 = 0$

$$\Leftrightarrow \cos x = \frac{1}{2} \Leftrightarrow x = \pm\frac{\pi}{3} + k2\pi$$

$+t = \sqrt{2} : \frac{\cos x}{\sin^2 x} = \sqrt{2} \Leftrightarrow \cos x = \sqrt{2}(1-\cos^2 x) \Leftrightarrow \sqrt{2}\cos^2 x + \cos x - \sqrt{2} = 0$

$$\Leftrightarrow \cos x = \frac{\sqrt{2}}{2} \Leftrightarrow x = \pm\frac{\pi}{4} + k2\pi$$

Vậy,phương trình có nghiệm: $x = \pm\frac{\pi}{3} + k2\pi, x = \pm\frac{\pi}{4} + k2\pi$

Bài 11. $\frac{4\sin^2 2x + 6\sin^2 x - 9 - 3\cos 2x}{\cos x} = 0$ (*)

Điều kiện: $\cos x \neq 0 \Leftrightarrow x \neq \frac{\pi}{2} + k\pi$

$(*) \Leftrightarrow 4(1-\cos^2 2x) + 3(1-\cos 2x) - 9 - 3\cos x = 0 \Leftrightarrow 4\cos^2 2x + 6\cos x + 2 = 0$

$$\Leftrightarrow \left[\begin{array}{l} \cos 2x = -1 \\ \cos 2x = -\frac{1}{2} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{2} + k\pi \\ x = \pm\frac{\pi}{3} + k\pi \end{array}\right.$$

Vậy,phương trình có nghiệm: $x=\pm\frac{\pi}{3}+k\pi$

Bài 12. $\cos x+\cos 3x+2\cos 5x=0 \Leftrightarrow (\cos 5x+\cos x)+(\cos 5x+\cos 3x)=0$

$$\Leftrightarrow 2\cos 3x\cos 2x+2\cos 4x\cos x=0$$

$$\Leftrightarrow (4\cos^3 x-3\cos x)\cos 2x+(2\cos^2 2x-1)\cos x=0$$

$$\Leftrightarrow \cos x[(4\cos^2 x-3)\cos 2x+2\cos^2 2x-1]=0$$

$$\Leftrightarrow \cos x\{[2(1+\cos 2x)-3]\cos 2x+2\cos^2 2x-1\}=0$$

$$\Leftrightarrow \cos x(4\cos^2 2x-\cos 2x-1)=0$$

$$\Leftrightarrow \left[\begin{array}{l}\cos x=0\\ \cos x=\frac{1-\sqrt{17}}{8}\\ \cos x=\frac{1+\sqrt{17}}{8}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{2}+k\pi\\ x=\pm\arccos\frac{1-\sqrt{17}}{8}+k2\pi\\ x=\pm\arccos\frac{1+\sqrt{17}}{8}+k2\pi\end{array}\right.$$

Bài 13. $\sin^8 x+\cos^8 x=\frac{17}{16}\cos^2 2x \qquad (*)$

$$\sin^8 x+\cos^8 x=(\sin^4 x+\cos^4 x)^2-2\sin^4 x\cos^4 x$$

$$=[(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x)]^2-\frac{1}{8}\sin^4 2x$$

$$=(1-\frac{1}{2}\sin^2 2x)^2-\frac{1}{8}\sin^4 2x \ =1-\sin^2 2x+\frac{1}{8}\sin^4 2x$$

$$(*)\Leftrightarrow 16(1-\sin^2 2x+\frac{1}{8}\sin^4 2x)=17(1-\sin^2 2x) \Leftrightarrow 2\sin^4 2x+\sin^2 2x-1=0$$

$$\Leftrightarrow \sin^2 2x=\frac{1}{2} \Leftrightarrow 1-2\sin^2 2x=0 \Leftrightarrow \cos 4x=0 \Leftrightarrow x=\frac{\pi}{8}+k\frac{\pi}{4}$$

Bài 14. $\sin\frac{5x}{2}=5\cos^3 x\sin\frac{x}{2} \qquad (*)$

Ta thấy: $\cos\frac{x}{2}=0 \Leftrightarrow x=\pi+k2\pi \Leftrightarrow \cos x=-1$

Thay vào phương trình (*) ta được:

$\sin(\frac{5\pi}{2}+5k\pi)=-\sin(\frac{\pi}{2}+k\pi)$ không thỏa mãn với mọi k

Do đó $\cos\frac{x}{2}$ không là nghiệm của phương trình nên:

$$(*)\Leftrightarrow \sin\frac{5x}{2}\cos\frac{x}{2}=5\cos^3x\sin\frac{x}{2}\cos\frac{x}{2}\Leftrightarrow \frac{1}{2}(\sin 3x+\sin 2x)=\frac{5}{2}\cos^3x\sin x$$

$$\Leftrightarrow 3\sin x-4\sin^3x+2\sin x\cos x-5\cos^3x\sin x=0$$

$$\Leftrightarrow \sin x(3-4\sin^2x+2\cos x-5\cos^3x)=0$$

$$\Leftrightarrow \sin x(5\cos^3x-4\cos^2x-2\cos x+1)=0$$

$$\Leftrightarrow \left[\begin{array}{l}\sin x=0\\ \cos x=1\\ \cos x=\frac{-1+\sqrt{21}}{10}\\ \cos x=\frac{-1-\sqrt{21}}{10}\end{array}\right.\Leftrightarrow \left[\begin{array}{l}x=k\pi\\ x=k2\pi\\ x=\pm\arccos\frac{-1+\sqrt{21}}{10}+k2\pi\\ x=\pm\arccos\frac{-1-\sqrt{21}}{10}+k2\pi\end{array}\right.$$

Vậy,phương trình có nghiệm: $x=k2\pi$, $x=\pm\arccos\frac{-1+\sqrt{21}}{10}+k2\pi$

$$x=\pm\arccos\frac{-1-\sqrt{21}}{10}+k2\pi$$

Bài 15. $\sin 2x(\cot x+\tan 2x)=4\cos^2x$ (1)

Điều kiện: $\begin{cases}\sin x\neq 0\\ \cos 2x\neq 0\end{cases}\Leftrightarrow\begin{cases}x\neq k\pi\\ x\neq\frac{\pi}{4}+k\frac{\pi}{2}\end{cases}$

Ta có: $\cot x+\tan 2x=\frac{\cos x}{\sin x}+\frac{\sin 2x}{\cos 2x}=\frac{\cos 2x\cos x+\sin 2x\sin x}{\sin x\cos 2x}=\frac{\cos x}{\sin x\cos 2x}$

$$(1)\Leftrightarrow 2\sin x\cos x\frac{\cos x}{\sin x\cos 2x}=4\cos^2x$$

$$\Leftrightarrow \frac{\cos^2 x}{\cos 2x} = 2\cos^2 x \Leftrightarrow \cos^2 x(1-2\cos 2x) = 0$$

$$\Leftrightarrow \left[\begin{array}{l} \cos x = 0 \\ \cos 2x = 1/2 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{2} + k\pi \\ x = \pm\frac{\pi}{6} + k\pi \end{array}\right.$$

Vậy,phương trình có nghiệm: $x = \frac{\pi}{2} + k\pi$, $x = \pm\frac{\pi}{6} + k\pi$

Bài 16. $2\cos^2\frac{6x}{5} + 1 = 3\cos\frac{8x}{5}$

$$\Leftrightarrow (1+\cos\frac{12x}{5}) + 1 = 2(2\cos^2\frac{4x}{5} - 1) \quad \Leftrightarrow 2 + 4\cos^3\frac{4x}{5} - 3\cos\frac{4x}{5} = 2(2\cos^2\frac{4x}{5} - 1)$$

Đặt: $t = \cos\frac{4x}{5}, -1 \le t \le 1$ phương trình trở thành:

$$4t^3 - 6t^2 - 3t - 5 = 0 \Leftrightarrow \left[\begin{array}{l} t = 1 \\ t = \frac{1-\sqrt{21}}{4} \end{array}\right.$$

$+\cos\frac{4x}{5} = 1 \Leftrightarrow x = k\frac{5\pi}{2}$

$+\cos\frac{4x}{5} = \frac{1-\sqrt{21}}{4} \Leftrightarrow x = \pm\frac{5}{4}\arccos\frac{1-\sqrt{21}}{4} + k\frac{5\pi}{2}$

Vậy,phương trình có nghiệm: $x = k\frac{5\pi}{2}$, $x = \pm\frac{5}{4}\arccos\frac{1-\sqrt{21}}{4} + k\frac{5\pi}{2}$

Bài 17. $\tan^3(x-\frac{\pi}{4}) = \tan x - 1 \quad (1)$

Điều kiện: $\left\{\begin{array}{l} \cos x \ne 0 \\ \cos(x-\frac{\pi}{4}) \ne 0 \end{array}\right. \Leftrightarrow \left\{\begin{array}{l} x \ne \frac{\pi}{2} + k\pi \\ x \ne \frac{3\pi}{4} + k\pi \end{array}\right.$

$$(1) \Leftrightarrow \frac{(\tan x-1)^3}{(1+\tan x)^3} = \tan x - 1 \Leftrightarrow (\tan x-1)^3 = (\tan x-1)(1+\tan x)^3$$

$$\Leftrightarrow (\tan x-1)[(1+\tan x)^3 - (\tan x-1)^2] = 0$$

$$\Leftrightarrow (\tan x-1)(\tan^3 x + 2\tan^2 x + 5\tan x) = 0$$

$$\Leftrightarrow \tan x(\tan x-1)(\tan^2 x + 2\tan x + 5) = 0$$

$$\Leftrightarrow \left[\begin{array}{l} \tan x = 0 \\ \tan x = 1 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = k\pi \\ x = \dfrac{\pi}{4} + k\pi \end{array}\right.$$

C2: Đặt: $t = x - \frac{\pi}{4}$

Bài 18. $\dfrac{\sin^4 2x + \cos^4 2x}{\tan(\frac{\pi}{4} - x)\tan(\frac{\pi}{4} + x)} = \cos^4 4x \qquad (1)$

Điều kiện: $\begin{cases} \sin(\frac{\pi}{4} - x)\cos(\frac{\pi}{4} - x) \neq 0 \\ \sin(\frac{\pi}{4} + x)\cos(\frac{\pi}{4} + x) \neq 0 \end{cases} \Leftrightarrow \begin{cases} \sin(\frac{\pi}{4} - 2x) \neq 0 \\ \sin(\frac{\pi}{4} + 2x) \neq 0 \end{cases} \Leftrightarrow \cos 2x \neq 0$

$$\tan(\frac{\pi}{4} - x)\tan(\frac{\pi}{4} + x) = \frac{1-\tan x}{1+\tan x}.\frac{1+\tan x}{1-\tan x} = 1$$

$$(1) \Leftrightarrow \sin^4 2x + \cos^4 2x = \cos^4 4x \Leftrightarrow 1 - 2\sin^2 2x\cos^2 2x = \cos^4 4x$$

$$\Leftrightarrow 1 - \frac{1}{2}\sin^2 4x = \cos^4 4x \Leftrightarrow 1 - \frac{1}{2}(1-\cos^2 4x) = \cos^4 4x$$

$$\Leftrightarrow 2\cos^4 4x - \cos^2 4x - 1 = 0 \Leftrightarrow \cos^2 4x = 1$$

$$\Leftrightarrow 1 - \cos^2 4x = 0 \Leftrightarrow \sin 4x = 0 \Leftrightarrow x = k\frac{\pi}{4}$$

Vậy,phương trình có nghiệm: $x = k\frac{\pi}{2}$

Bài 19. $48-\dfrac{1}{\cos^4 x}-\dfrac{2}{\sin^2 x}(1+\cot 2x\cot x)=0$ (*)

Điều kiện: $\sin 2x \neq 0 \Leftrightarrow x \neq k\dfrac{\pi}{2}$

Ta có: $1+\cot 2x\cot x = 1+\dfrac{\cos 2x\cos x}{\sin 2x\sin x} = \dfrac{\cos 2x\sin x+\sin 2x\sin x}{\sin 2x\cos x}$

$$=\dfrac{\cos x}{2\sin^2 x\cos x} = \dfrac{1}{2\sin^2 x}$$

$$(*) \Leftrightarrow 48-\dfrac{1}{\cos^4 x}-\dfrac{1}{\sin^4 x}=0 \Leftrightarrow 48=\dfrac{1}{\cos^4 x}+\dfrac{1}{\sin^4 x}$$

$$\Leftrightarrow 48\sin^4 x\cos^4 x=\sin^4 x+\cos^4 x \Leftrightarrow 3\sin^4 2x = 1-\dfrac{1}{2}\sin^2 2x$$

$$\Leftrightarrow 6\sin^4 2x+\sin^2 2x-2=0 \Leftrightarrow \sin^2 2x=\dfrac{1}{2} \Leftrightarrow 1-2\sin^2 2x=0$$

$$\Leftrightarrow \cos 4x=0 \Leftrightarrow x=\dfrac{\pi}{8}+k\dfrac{\pi}{4}$$

Vậy,phương trình có nghiệm: $x=\dfrac{\pi}{8}+k\dfrac{\pi}{4}$

Bài 20. $\sin^8 x+\cos^8 x = 2(\sin^{10} x+\cos^{10} x)+\dfrac{5}{4}\cos 2x$

$$\Leftrightarrow \sin^8 x(1-2\sin^2 x)-\cos^8 x(2\cos^2 x-1)=\dfrac{5}{4}\cos 2x$$

$$\Leftrightarrow \sin^8 x\cos 2x-\cos^8 x\cos 2x=\dfrac{5}{4}\cos 2x$$

$$\Leftrightarrow 4\cos 2x(\cos^8 x-\sin^8 x)+5\cos 2x=0$$

$$\Leftrightarrow 4\cos 2x(\cos^4 x-\sin^4 x)(\cos^4 x+\sin^4 x)+5\cos 2x=0$$

$$\Leftrightarrow 4\cos 2x(\cos^2 x-\sin^2 x)(\cos^2 x+\sin^2 x)(\cos^4 x+\sin^4 x)+5\cos 2x=0$$

$$\Leftrightarrow 4\cos 2x(\cos^2 x-\sin^2 x)(1-\dfrac{1}{2}\sin^2 2x)+5\cos 2x=0$$

$$\Leftrightarrow 4\cos^2 2x(1-\frac{1}{2}\sin^2 2x)+5\cos 2x=0$$

$$\Leftrightarrow 4\cos 2x(4\cos 2x-2\cos 2x\sin^2 2x+5)=0$$

$$\Leftrightarrow 4\cos 2x[4\cos 2x-2\cos 2x(1-\cos^2 2x)+5]=0$$

$$\Leftrightarrow 4\cos 2x(2\cos^3 2x+2\cos 2x+5)=0 \Leftrightarrow \cos 2x=0 \Leftrightarrow x=\frac{\pi}{4}+k\frac{\pi}{2}$$

MỘT SỐ KĨ NĂNG KẾT HỢP NGHIỆM VÀ ĐIỀU KIỆN TRONG PHƯƠNG TRÌNH LƯỢNG GIÁC CÓ ĐIỀU KIỆN

I. CÁC PHƯƠNG PHÁP KẾT HỢP NGHIỆM VỚI ĐIỀU KIỆN PHỔ BIẾN:

1. Biểu diễn nghiệm (của phương trình hệ quả) và điều kiện (của phương trình ban đầu) qua cùng một hàm số lượng giác:

1.1 Kiến thức cơ sở:

Trong phần này cần sử dụng tốt các công thức sau:

Công thức nhân đôi

Công thức hạ bậc

Các hằng đẳng thức cơ bản của lượng giác

Từ đó ta có các kết quả cần chú ý sau

$sin2a=0 \Leftrightarrow \left[\begin{array}{l}\sin a=0\\ \cos a=0\end{array}\right.$ $\qquad$ $sin2a\neq 0 \Leftrightarrow \begin{cases}\sin a\neq 0\\ \cos a\neq 0\end{cases}$

$\sin^2 a=0 \Leftrightarrow \cos a=\pm 1;$ $\qquad$ $\sin^2 a=1 \Leftrightarrow \cos a=0$

$\cos^2 a=0 \Leftrightarrow \sin a=\pm 1;$ $\qquad$ $\cos^2 a=1 \Leftrightarrow \sin a=0$

$\sin a \neq 0 \Leftrightarrow \cos a \neq \pm 1$; $\cos a \neq 0 \Leftrightarrow \sin a \neq \pm 1$

1.2 Một số ví dụ minh hoạ:

Ví dụ 1: (Đề thi tuyển sinh ĐH – CĐ 2010, khối A)

Giải phương trình

$$\frac{(1+\sin x+\cos 2x)\sin\left(x+\frac{\pi}{4}\right)}{1+\tan x}=\frac{1}{\sqrt{2}}\cos x$$

Lời giải:

Điều kiện: $\begin{cases}\cos x \neq 0\\ \tan x \neq -1\end{cases} \Leftrightarrow \begin{cases}\sin x \neq \pm 1\\ \tan x \neq -1\end{cases}$

Khi đó $\dfrac{(1+\sin x+\cos 2x)\sin\left(x+\frac{\pi}{4}\right)}{1+\tan x}=\dfrac{1}{\sqrt{2}}\cos x$

$$\Rightarrow \cos x(1+\sin x+\cos 2x)\sqrt{2}.\sin\left(x+\frac{\pi}{4}\right)=\cos x(\sin x+\cos x)$$

$$\Leftrightarrow (1+\sin x+\cos 2x)\sqrt{2}.\sin\left(x+\frac{\pi}{4}\right)=(\sin x+\cos x) \quad (\text{do } \cos x \neq 0)$$

$$\Leftrightarrow (\sin x+\cos x)(\sin x+\cos 2x)=0 \Leftrightarrow (\sin x+\cos x)\left(\sin x+1-2\sin^2 x\right)=0$$

$$\Leftrightarrow \left[\begin{array}{l}\sin x+\cos x=0\\ 2\sin^2 x-\sin x-1=0\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\sin x=-\cos x\\ \sin x=1\\ \sin x=-\frac{1}{2}\end{array}\right. \Leftrightarrow \left[\begin{array}{ll}\tan x=-1 & (L)\\ \sin x=1 & (L)\\ \sin x=-\frac{1}{2} & (t/m)\end{array}\right.$$

$$\Leftrightarrow \sin x=-\frac{1}{2} \Leftrightarrow \left[\begin{array}{l}x=-\frac{\pi}{6}+k.2\pi\\ x=\frac{7\pi}{6}+k.2\pi\end{array}\right. \quad (k \in Z)$$

Ví dụ 2: (Đề thi tuyển sinh ĐH – CĐ 2006, khối B)

Giải phương trình $\cot x + \sin x\left(1 + \tan x.\tan\frac{x}{2}\right) = 4$

Lời giải: Điều kiện $\begin{cases} \cos x \neq 0 \\ \sin x \neq 0 \\ \cos\frac{x}{2} \neq 0 \end{cases} \Leftrightarrow \sin 2x \neq 0$

Ta có $\cot x + \sin x\left(1 + \tan x.\tan\frac{x}{2}\right) = 4 \Leftrightarrow \frac{\cos x}{\sin x} + \sin x\left(1 + \frac{\sin x}{\cos x}.\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right) = 4$

$$\Leftrightarrow \frac{\cos x}{\sin x} + \sin x\left(\frac{\cos x.\cos\frac{x}{2} + \sin x.\sin\frac{x}{2}}{\cos x.\cos\frac{x}{2}}\right) = 4 \Leftrightarrow \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} = 4$$

$$\Leftrightarrow \frac{2}{\sin 2x} = 4 \Leftrightarrow \sin 2x = \frac{1}{2} \qquad (t/m)$$

$$\Leftrightarrow \left[\begin{array}{l} 2x = \frac{\pi}{6} + k.2\pi \\ 2x = \frac{5\pi}{6} + k.2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{12} + k.\pi \\ x = \frac{5\pi}{12} + k.\pi \end{array}\right. \qquad (k \in Z)$$

Ví dụ 3: Giải phương trình

$$\frac{1}{\cos x} + \frac{1}{\sin 2x} = \frac{2}{\sin 4x}$$

Lời giải:

Điều kiện $\begin{cases}\cos x \neq 0\\ \sin 2x \neq 0\\ \sin 4x \neq 0\end{cases} \Leftrightarrow \begin{cases}\sin x \neq \pm 1\\ \sin x \neq 0\\ \cos 2x \neq 0\end{cases} \Leftrightarrow \begin{cases}\sin x \neq \pm 1\\ \sin x \neq 0\\ 1-2\sin^2 x \neq 0\end{cases} \Leftrightarrow \begin{cases}\sin x \neq \pm 1\\ \sin x \neq 0\\ \sin x \neq \pm \dfrac{\sqrt{2}}{2}\end{cases}$

Khi đó $\dfrac{1}{\cos x}+\dfrac{1}{\sin 2x}=\dfrac{2}{\sin 4x}$

$$\Rightarrow 4\sin x.\cos 2x+2\cos 2x=2 \Leftrightarrow \sin x\left(2\sin^2 x+\sin x-1\right)=0 \Leftrightarrow \left[\begin{array}{l}\sin x=-1\\ \sin x=0\\ \sin x=\dfrac{1}{2}\end{array}\right.$$

Đối chiếu với điều kiện ta được $\sin x=\dfrac{1}{2} \Leftrightarrow \left[\begin{array}{l}x=\dfrac{\pi}{6}+k.2\pi\\ x=\dfrac{5\pi}{6}+k.2\pi\end{array}\right. \quad (k\in Z)$

Vậy phương trình có nghiệm là $\left[\begin{array}{l}x=\dfrac{\pi}{6}+k.2\pi\\ x=\dfrac{5\pi}{6}+k.2\pi\end{array}\right. \quad (k\in Z)$

<u>Ví dụ 4:</u> Giải phương trình $\dfrac{\sin^4 2x+\cos^4 2x}{\tan\left(\dfrac{\pi}{4}-x\right)\tan\left(\dfrac{\pi}{4}+x\right)}=\cos^4 4x$

Lời giải:

Điều kiện $\begin{cases}\sin\left(\frac{\pi}{4}-x\right)\neq 0\\ \cos\left(\frac{\pi}{4}-x\right)\neq 0\\ \sin\left(\frac{\pi}{4}+x\right)\neq 0\\ \cos\left(\frac{\pi}{4}+x\right)\neq 0\end{cases}\Leftrightarrow\begin{cases}\sin\left(\frac{\pi}{2}-2x\right)\neq 0\\ \sin\left(\frac{\pi}{2}+2x\right)\neq 0\end{cases}\Leftrightarrow \cos 2x\neq 0\Leftrightarrow \sin 2x\neq \pm 1$

Nhận thấy $\tan\left(\frac{\pi}{4}-x\right).\tan\left(\frac{\pi}{4}+x\right)=1$, do đó phương trình đã cho trở thành

$$\sin^4 2x+\cos^4 2x=\cos^4 4x\Leftrightarrow 1-\frac{1}{2}\sin^2 4x=\cos^4 4x\Leftrightarrow 2\cos^4 4x-\cos^2 4x-1=0$$

$$\Leftrightarrow \cos^2 4x=1\Leftrightarrow \sin 4x=0\Leftrightarrow \left[\begin{array}{l}\sin 2x=0\\ \cos 2x=0\end{array}\right.$$

Đối chiếu điều kiện ta được $\sin 2x=0\Leftrightarrow x=k\frac{\pi}{2}\quad (k\in Z)$

Ví dụ 5: Giải phương trình $\frac{\sin^2 2x+\cos^4 2x-1}{\sqrt{\sin x.\cos x}}=0$

Lời giải: Điều kiện $\sin 2x>0$. Khi đó phương trình đã cho trở thành

$$\sin^2 2x+\cos^4 2x-1=0\Leftrightarrow \cos^4 2x-\cos^2 2x=0\Leftrightarrow \left[\begin{array}{l}\cos^2 2x=0\\ \cos^2 2x=1\end{array}\right.\Leftrightarrow \left[\begin{array}{l}\sin 2x=\pm 1\\ \sin 2x=0\end{array}\right.$$

Đối chiếu điều kiện ta được $\sin 2x=1\Leftrightarrow 2x=\frac{\pi}{2}+k.2\pi\Leftrightarrow x=\frac{\pi}{4}+k.\pi\quad (k\in Z)$

Các bài tập tương tự

1/ $\cos 2x-\tan^2 x=\frac{\cos^2 x-\cos^3 x-1}{\cos^2 x}$; 2/ $\cot x-1=\frac{\cos 2x}{1+\tan x}+\sin^2 x-\frac{1}{2}\sin 2x$ (2003_A);

3/ $\cot x-\tan x+4\sin 2x=\frac{2}{\sin 2x}$ (2003_B); **4/** $\sin^2\left(\frac{x}{2}-\frac{\pi}{4}\right)\tan^2 x-\cos^2\frac{x}{2}=0$ (2003_D);

5/ $5\sin x-2=3(1-\sin x)\tan^2 x$ (2004_B).

2. Thử trực tiếp và xét mệnh đề đối lập

2.1 Kiến thức cơ sở

+ Các nhận xét về tính chu kì của hàm số lượng giác.

$\sin(\alpha+k2\pi)=\sin\alpha \quad \forall\alpha$; $\quad cos(\alpha+k2\pi)=cos\alpha \quad \forall\alpha$;

$\tan(\alpha+k\pi)=\tan\alpha \quad \forall\alpha$; $\quad \cot(\alpha+k\pi)=\cot\alpha \quad \forall\alpha$

+ Các công thức về giá trị lượng giác của các cung có liên quan đặc biệt.

2.2 Một số ví dụ minh hoạ

<u>Ví dụ 1:</u> Giải phương trình $cos3x.\tan 5x=\sin 7x$

Lời giải: Điều kiện $cos5x\neq 0$

Khi đó phương trình đã cho trở thành

$$2\sin 5x.cos3x=2\sin 7x.cos5x \Leftrightarrow \sin 8x=\sin 12x \Leftrightarrow \left[\begin{array}{l} x=\frac{k\pi}{2} \\ x=\frac{\pi}{20}+\frac{k\pi}{10} \end{array}\right. \quad (k\in Z)$$

Với $x=\frac{k\pi}{2}$ thì $cos5x=cos\frac{5k\pi}{2}=cos\left(\frac{k\pi}{2}+k2\pi\right)=cos\left(\frac{k\pi}{2}\right)\neq 0 \Leftrightarrow k=2m \quad (m\in Z)$

Với $x=\frac{\pi}{20}+\frac{k\pi}{10}$ thì $cos5x=cos\left(\frac{\pi}{4}+\frac{k\pi}{2}\right)\neq 0$

Vậy phương trình đã cho có nghiệm là $x=m\pi$; $\quad x=\frac{\pi}{20}+\frac{k\pi}{10} \quad (m,k\in Z)$

<u>Ví dụ 2:</u> (Đề thi tuyển sinh ĐH – CĐ, 2011, khối A)

Giải phương trình $\dfrac{1+\sin 2x+\cos 2x}{1+\cot^2 x}=\sqrt{2}\sin x\sin 2x$

Lời giải: Điều kiện $\sin x\neq 0\Leftrightarrow \cos x\neq \pm 1$

Khi đó phương trình đã cho trở thành

$$\sin^2 x(1+\sin 2x+\cos 2x)=2\sqrt{2}\sin^2 x.\cos x\Leftrightarrow 1+2\sin x.\cos x+2\cos^2 x-1=2\sqrt{2}\cos x$$

$$\Leftrightarrow 2\cos x\left(\sin x+\cos x-\sqrt{2}\right)=0\Leftrightarrow \left[\begin{array}{l}\cos x=0\quad (t/m)\\ \sin x+\cos x=\sqrt{2}\quad (*)\end{array}\right.$$

Giả sử $\sin x=0\Leftrightarrow \cos x=\pm 1$, khi đó $(*)\Leftrightarrow 0\pm 1=\sqrt{2}$ (vô lí)

Do đó phương trình tương đương với $\left[\begin{array}{l}\cos x=0\\ \cos\left(x-\dfrac{\pi}{4}\right)=1\end{array}\right.\Leftrightarrow \left[\begin{array}{l}x=\dfrac{\pi}{2}+k\pi\\ x=\dfrac{\pi}{4}+k2\pi\end{array}\right.$

Vậy phương trình có nghiệm là $\left[\begin{array}{l}x=\dfrac{\pi}{2}+k\pi\\ x=\dfrac{\pi}{4}+k2\pi\end{array}\right.\quad (k\in Z)$

<u>Ví dụ 3:</u> Giải phương trình $3\sin x+2\cos x=3(1+\tan x)-\dfrac{1}{\cos x}$

Lời giải: Điều kiện $\cos x\neq 0\Leftrightarrow \sin x\neq \pm 1$

Khi đó

$$3\sin x+2\cos x=3(1+\tan x)-\frac{1}{\cos x}\Leftrightarrow \cos x(3\sin x+2\cos x)=3(\cos x+\sin x)-1$$

$$\Leftrightarrow \cos x(3\sin x+2\cos x)-\cos x=3\sin x+2\cos x-1$$

$$\Leftrightarrow \cos x(3\sin x+2\cos x-1)-(3\sin x+2\cos x-1)=0$$

$$\Leftrightarrow (3\sin x+2\cos x-1)(\cos x-1)=0\Leftrightarrow \left[\begin{array}{ll}\cos x-1=0 & (1)\\ 3\sin x+2\cos x-1=0 & (2)\end{array}\right.$$

$(1)\Leftrightarrow \cos x=1$ thoả mãn điều kiện, do đó ta được $x=k2\pi,\quad k\in Z$

Tiếp theo giả sử $cosx = 0 \Leftrightarrow \sin x = \pm 1$, thay vào (2) ta được $\pm 3 - 1 = 0$ (vô lí)

Tức là các nghiệm của (2) đều thoả mãn điều kiện.

Giải (2) ta được $x = \alpha \pm \arccos\frac{1}{\sqrt{13}} + k2\pi \quad k \in Z$,

(với $\cos\alpha = \frac{2}{\sqrt{13}}; \quad \sin\alpha = \frac{3}{\sqrt{13}}$)

Vậy phương trình có nghiệm $\left[\begin{array}{l} x = k2\pi \\ x = \alpha \pm \arccos\frac{1}{\sqrt{13}} + k2\pi \end{array}\right. \quad k \in Z$

<u>Ví dụ 4:</u> Giải phương trình

$$\frac{\tan^2 x + \tan x}{\tan^2 x + 1} = \frac{\sqrt{2}}{2}\sin\left(x + \frac{\pi}{4}\right)$$

Lời giải: Điều kiện $cosx \neq 0 \Leftrightarrow \sin x \neq \pm 1$

$$\frac{\tan^2 x + \tan x}{\tan^2 x + 1} = \frac{\sqrt{2}}{2}\sin\left(x + \frac{\pi}{4}\right) \Leftrightarrow \cos^2 x\left(\tan^2 x + \tan x\right) = \frac{\sqrt{2}}{2}\left(\frac{\sqrt{2}}{2}\sin x + \frac{\sqrt{2}}{2}\cos x\right)$$

Khi đó $\Leftrightarrow \sin^2 x + \cos x.\sin x = \frac{1}{2}(\sin x + \cos x) \Leftrightarrow 2\sin x(\sin x + \cos x) - (\sin x + \cos x) = 0$

$\Leftrightarrow (\sin x + \cos x)(2\sin x - 1) = 0 \qquad (*)$

Giả sử $cosx = 0 \Leftrightarrow \sin x = \pm 1$, thay vào (*) ta được $\pm 1(\pm 2 - 1) = 0$ (vô lí)

Tức là các nghiệm của (*) đều thoả mãn điều kiện.

Giải (*) ta được $x = \frac{3\pi}{4} + k\pi; \quad x = \frac{\pi}{6} + k2\pi; \quad x = \frac{5\pi}{6} + k2\pi \qquad (k \in Z)$

<u>Ví dụ 5:</u> Giải phương trình

$$\tan 5x.\tan 2x = 1$$

Lời giải: Điều kiện $\begin{cases}\cos 5x \neq 0\\ \cos 2x \neq 0\end{cases} \Leftrightarrow \begin{cases} x \neq \dfrac{\pi}{10} + m\dfrac{\pi}{5} & (1)\\ x \neq \dfrac{\pi}{4} + n\dfrac{\pi}{2} & (2)\end{cases} \quad (m, n \in Z)$

phương trình tương đương với

$$\tan 5x = \frac{1}{\tan 2x} \Leftrightarrow \tan 5x = \cot 2x \Leftrightarrow x = \frac{\pi}{14} + k\frac{\pi}{7} \quad (k \in Z)$$

+ Đối chiếu điều kiện (1)

Giả sử $\dfrac{\pi}{14} + k\dfrac{\pi}{7} = \dfrac{\pi}{10} + m\dfrac{\pi}{5} \Leftrightarrow k = m + \dfrac{1+2m}{5}$

Do $k, m \in Z$ nên $\exists t \in Z:\ t = \dfrac{1+2m}{5} \Leftrightarrow m = 2t + \dfrac{t-1}{2}$

Lại do $t, m \in Z$ nên $\exists s \in Z:\ s = \dfrac{t-1}{2} \Leftrightarrow t = 2s + 1$

Từ đó $k = 7s + 3$. Suy ra $x = \dfrac{\pi}{14} + k\dfrac{\pi}{7}$ với $k \neq 7s + 3$ thoả mãn phương trình

+ Đối chiếu điều kiện (2)

Giả sử $\dfrac{\pi}{14} + k\dfrac{\pi}{7} = \dfrac{\pi}{4} + n\dfrac{\pi}{2} \Leftrightarrow 4k - 14n = 5 \quad (3)$

Ta thấy vế trái của (3) chẵn, vế phải của (3) lẻ nên không tồn tại $k, n \in Z$ thoả mãn (3).

Từ đó suy ra điều kiên (2) luôn được thoả mãn.

Vậy phương trình đã cho có nghiệm là $x = \dfrac{\pi}{14} + k\dfrac{\pi}{7} \quad (k \in Z)$

Các bài tập tương tự

1/ $\dfrac{1}{\tan x + \cot 2x} = \dfrac{\sqrt{2}(\sin x - \cos x)}{\cot x - 1}$; 2/ $2\sin x + \cot x = 2\sin 2x + 1$;

3/ $\frac{\sin x.\cot 5x}{\cos 9x}=1;$ 4/ $\tan^4 x+1=\frac{\left(2-\sin^2 2x\right).\sin 3x}{\cos^4 x};$

5/ $\left(4\cos^2 x-3\right)\sin 3x=\frac{1}{2}.$

3. Biểu diễn trên đường tròn lượng giác (ĐTLG)

3.1 Kiến thức cơ sở

+ Mỗi cung (hoặc góc) lượng giác được biểu diễn bởi một điểm trên ĐTLG

$x=\alpha+k2\pi$ được biểu diễn bởi một điểm trên ĐTLG;

$x=\alpha+k\pi$ được biểu diễn trên ĐTLG bởi 2 điểm đối xứng nhau qua O;

$x=\alpha+\frac{k2\pi}{3}$ được biểu diễn trên ĐTLG bởi 3 điểm cách đều nhau, tạo thành 3 đỉnh một tam giác đều nội tiếp ĐTLG;

$x=\alpha+\frac{k2\pi}{n}$ được biểu diễn trên ĐTLG bởi *n* điểm cách đều nhau, tạo thành *n* đỉnh một đa giác đều nội tiếp ĐTLG.

+ Ta biểu diễn trên ĐTLG những điểm không thoả mãn điều kiện (đánh dấu "x") và những điểm nghiệm tìm được (đánh dấu "o"). Những điểm đánh dấu "o" mà không trùng với những điểm đánh dấu "x" chính là những điểm thoả mãn điều kiện.

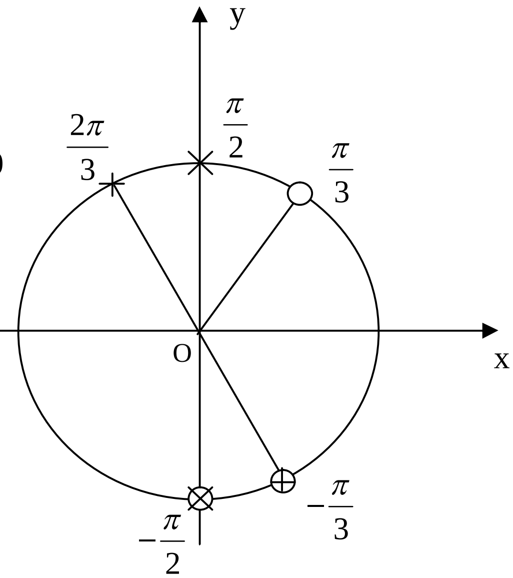
3.2 Một số ví dụ minh hoạ

Ví dụ 1: (Đề thi tuyển sinh ĐH – CĐ 2011, khối D)

Giải phương trình

$$\frac{\sin 2x+2\cos x-\sin x-1}{\tan x+\sqrt{3}}=0$$

Lời giải: Điều kiện $\begin{cases} \tan x \neq -\sqrt{3} \\ \cos x \neq 0 \end{cases} \Leftrightarrow \begin{cases} x \neq -\dfrac{\pi}{3} + m\pi \\ x \neq \dfrac{\pi}{2} + n\pi \end{cases} \quad (m, n \in Z)$



Khi đó phươngtrình đã cho trở thành

$$\sin 2x + 2\cos x - \sin x - 1 = 0 \Leftrightarrow 2\cos x(\sin x + 1) - (\sin x + 1) = 0$$

$$\Leftrightarrow (\sin x + 1)(2\cos x - 1) = 0 \Leftrightarrow \left[\begin{array}{l} \sin x = -1 \\ \cos x = \dfrac{1}{2} \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = -\dfrac{\pi}{2} + k2\pi \\ x = \pm\dfrac{\pi}{3} + k2\pi \end{array}\right.$$

Kết hợp với điều kiện trên đường tròn lượng giác (như hình bên) ta được nghiệm của phương trình là

$$x = \frac{\pi}{3} + k2\pi \qquad (k \in Z)$$

<u>Ví dụ 2:</u> (Đề thi tuyển sinh ĐH – CĐ 2006, khối A)

Giải phương trình $\dfrac{2\left(\cos^6 x + \sin^6 x\right) - \sin x.\cos x}{\sqrt{2} - 2\sin x} = 0$

Lời giải: Điều kiện $\sin x \neq \dfrac{\sqrt{2}}{2} \Leftrightarrow \begin{cases} x \neq \dfrac{\pi}{4} + m2\pi \\ x \neq \dfrac{3\pi}{4} + n2\pi \end{cases} \quad (m, n \in Z)$

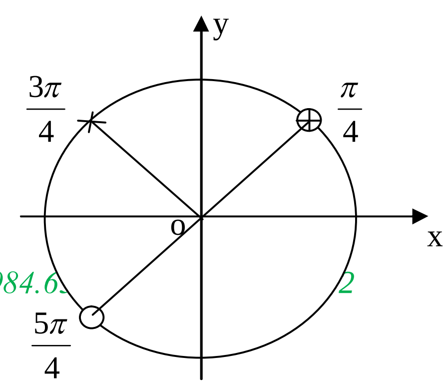
Khi đó phương trình đã cho trở thành

$$2\left(\cos^6 x + \sin^6 x\right) - \sin x.\cos x = 0$$

$$\Leftrightarrow 2\left(1 - \frac{3}{4}\sin^2 2x\right) - \frac{1}{2}\sin 2x = 0$$

$$\Leftrightarrow 3\sin^2 2x + \sin 2x - 4 = 0 \Leftrightarrow \sin 2x = 1$$

$$\Leftrightarrow x = \frac{\pi}{4} + k\pi \quad (k \in Z)$$



Kết hợp với điều kiện trên đường tròn lượng giác ta được nghiệm của phương trình là

$$x = \frac{5\pi}{4} + k2\pi \qquad (k \in Z)$$

Ví dụ 3: Giải phương trình $\dfrac{\sin x+\sin 2x}{\sin 3x}=-1$

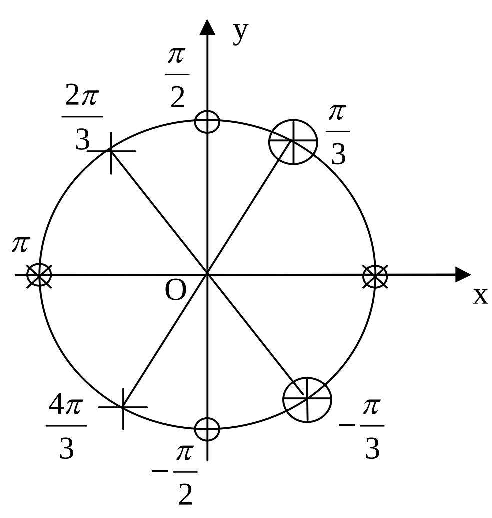
Lời giải: Điều kiện $\sin 3x\neq 0 \Leftrightarrow 3x\neq k\pi \Leftrightarrow x\neq k\dfrac{\pi}{3}$

Khi đó $\dfrac{\sin x+\sin 2x}{\sin 3x}=-1 \Leftrightarrow \sin x+\sin 2x+\sin 3x=0$

$\Leftrightarrow 2\sin 2x.\cos x+\sin 2x=0$

$$\Leftrightarrow \sin 2x\left(2\cos x+1\right)=0 \Leftrightarrow \left[\begin{array}{l}\sin 2x=0\\ \cos x=-\dfrac{1}{2}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=k\dfrac{\pi}{2}\\ x=\pm\dfrac{2\pi}{3}+k2\pi\end{array}\right.$$

Kết hợp với điều kiện trên đường tròn lượng giác .Ta được nghiệm của phương trình là $x=\dfrac{\pi}{2}+k\pi$.



Các bài tập tương tự

1/ $\dfrac{\sin x+\sin 2x+\sin 3x}{\cos x+\cos 2x+\cos 3x}=\sqrt{3}$; 2/ $\left|\cos x\right|+\sin 3x=0$;

3/ $\dfrac{\sqrt{1-\cos x}+\sqrt{1+\cos x}}{\cos x}=4\sin x$;

4/ $\dfrac{2\sqrt{3}.\cos^2 x+2\sin 3x.\cos x-\sin 4x-\sqrt{3}}{\sqrt{3}\sin x+\cos x}=1$;

5/ $\dfrac{\left(1-2\sin x\right)\cos x}{\left(1+2\sin x\right)\left(1-\sin x\right)}=\sqrt{3}$ (Tuyển sinh ĐH – CĐ 2009, khối A).

PHƯƠNG TRÌNH LƯỢNG GIÁC TRONG CÁC ĐỀ THI ĐẠI HỌC 2002 - 2011

Bai 1: [ĐH A02] Tìm $x \in (0;2\pi): 5\left(\sin x + \dfrac{\cos 3x + \sin 3x}{1 + 2\sin 2x}\right) = \cos 2x + 3$

Bai 2: [ĐH B02] $\sin^2 3x - \cos^2 4x = \sin^2 5x - \cos^2 6x$

Bai 3: [ĐH D02] Tìm $x \in [0;14]$: $\cos 3x - 4\cos 2x + 3\cos x - 4 = 0$

Bai 4: [Dự bị 1 ĐH02] Xác định m để phương trình sau có ít nhất 1 nghiệm thuộc $\left[0;\dfrac{\pi}{2}\right]$

$2\left(\sin^4 x + \cos^4 x\right) + \cos 4x + \sin 2x - m = 0$

Bai 5: [Dự bị 2 ĐH02] $\dfrac{\sin^4 x + \cos^4 x}{5\sin 2x} = \dfrac{1}{2}\cot 2x - \dfrac{1}{8\sin 2x}$

Bai 6: [Dự bị 3 ĐH02] $\tan^4 x + 1 = \dfrac{\left(2 - \sin^2 2x\right)\sin 3x}{\cos^4 x}$

Bai 7: [Dự bị 4 ĐH02] $\tan x + \cos x - \cos^2 x = \sin x\left(1 + \tan x.\tan\dfrac{x}{2}\right)$

Bai 8: [Dự bị 5 ĐH02] Cho phương trình : $\dfrac{2\sin x + \cos x + 1}{\sin x - 2\cos x + 3} = a$

a) Giải phương trình với $a=\dfrac{1}{3}$ b) Tìm a để phương trình trên có nghiệm.

Bai 9: [Dự bị 6 ĐH02] $\sqrt{\dfrac{1}{8\cos^2 x}} = \sin x$

Bai 10: [ĐH A03] $\cot x - 1 = \dfrac{\cos 2x}{1 + \tan x} + \sin^2 x - \dfrac{1}{2}\sin 2x$

Bai 11: [ĐH B03] $\cot x - \tan x + 4\sin 2x = \dfrac{2}{\sin 2x}$

Bai 12: [ĐH D03] $\sin^2\left(\dfrac{x}{2} - \dfrac{\pi}{4}\right)\tan^2 x - \cos^2\dfrac{x}{2} = 0$

Bai 13: [Dự bị 1 ĐH A03] $3 - \tan x\left(\tan x + 2\sin x\right) + 6\cos x = 0$

Bai 14: [Dự bị 2 ĐH A03] $\cos 2x + \cos x\left(2\tan^2 x - 1\right) = 2$

Bai 15: [Dự bị 1 ĐH B03] $3\cos 4x - 8\cos^6 x + 2\cos^2 x + 3 = 0$

Bai 16: [Dự bị 2 ĐH B03] $\dfrac{\left(2-\sqrt{3}\right)\cos x-2\sin^2\left(\dfrac{x}{2}-\dfrac{\pi}{4}\right)}{2\cos x-1}=1$

Bai 17: [Dự bị 1 ĐH D03] $\dfrac{\cos^2 x\left(\cos x-1\right)}{\sin x+\cos x}=2\left(1+\sin x\right)$

Bai 18: [Dự bị 2 ĐH D03] $\cot x=\tan x+\dfrac{2\cos 4x}{\sin 2x}$

Bai 19: [ĐH B04] $5\sin x-2=3(1-\sin x)\tan^2 x$

Bai 20: [ĐH D04] $\left(2\cos x-1\right)\left(2\sin x+\cos x\right)=\sin 2x-\sin x$

Bai 21: [Dự bị 1 ĐH A04] $\sin x+\sin 2x=\sqrt{3}\left(\cos x+\operatorname{cox}2x\right)$

Bai 22: [Dự bị 2 ĐH A04] $\sqrt{1-\sin x}+\sqrt{1-\cos x}=1$

Bai 23: [Dự bị 1 ĐH B04] $4\left(\sin^3 x+\cos^3 x\right)=\cos x+3\sin x$

Bai 24: [Dự bị 2 ĐH B04] $\dfrac{1}{\cos x}-\dfrac{1}{\sin x}=2\sqrt{2}\cos\left(x+\dfrac{\pi}{4}\right)$

Bai 25: [Dự bị 1 ĐH D04] $\sin 4x\sin 7x=\cos 3x\cos 6x$

Bai 26: [Dự bị 2 ĐH D04] $\sin 2x-2\sqrt{2}\left(\sin x+\cos x\right)-5=0$

Bai 27: [ĐH A05] $\cos^2 3x\cos 2x-\cos^2 x=0$

Bai 28: [ĐH B05] $1+\sin+\cos x+\sin 2x+\cos 2x=0$

Bai 29: [ĐH D05] $\cos^4 x+\sin^4 x+\cos\left(x-\dfrac{\pi}{4}\right)\sin\left(3x-\dfrac{\pi}{4}\right)-\dfrac{3}{2}=0$

Bai 30: [Dự bị 1 ĐH A05] Tìm $x\in\left(0;\pi\right)$ $4\sin^2\dfrac{x}{2}-\sqrt{3}\cos 2x=1+2\cos^2\left(x-\dfrac{3\pi}{4}\right)$

Bai 31: [Dự bị 2 ĐH A05] $2\sqrt{2}\cos^3\left(x-\dfrac{\pi}{4}\right)-3\cos x-\sin x=0$

Bai 32: [Dự bị 1 ĐH B05] $2\sqrt{2}\cos^3\left(x-\dfrac{\pi}{4}\right)-3\cos x-\sin x=0$

Bai 33: [Dự bị 2 ĐH B05] $\tan\left(\dfrac{\pi}{2}+x\right)-3\tan^2 x=\dfrac{\cos 2x-1}{\cos^2 x}$

Bai 34: [Dự bị 1 ĐH D05] $\tan\left(\frac{3\pi}{2}-x\right)+\frac{\sin x}{1+\cos x}=2$

Bai 35: [Dự bị 2 ĐH D05] $\sin 2x+\cos 2x+3\sin x-\cos x-2=0$

Bai 36: [ĐH A06] $\frac{2\left(\cos^6 x+\sin^6 x\right)-\sin x\cos x}{\sqrt{2}-2\sin x}=0$

Bai 37: [ĐH B06] $\cot x+\sin x\left(1+\tan x\tan\frac{x}{2}\right)=4$

Bai 38: [ĐH D06] $\cos 3x+\cos 2x-\cos x-1=0$

Bai 39: [Dự bị 1 ĐH A06] $\cos 3x\cos^3 x-\sin 3x\sin^3 x=\frac{2+3\sqrt{2}}{8}$

Bai 40: [Dự bị 2 ĐH A06] $2\sin\left(2x-\frac{\pi}{6}\right)+4\sin x+1=0$

Bai 41: [Dự bị 1 ĐH B06] $\left(2\sin^2 x-1\right)\tan^2 2x+3\left(2\cos^2 x-1\right)=0$

Bai 42: [Dự bị 2 ĐH B06] $\cos 2x+\left(1+2\cos x\right)\left(\sin x-\cos x\right)=0$

Bai 43: [Dự bị 1 ĐH D06] $\cos^3 x+\sin^3 x+2\sin^2 x=1$

Bai 44: [Dự bị 2 ĐH D06] $4\sin^3 x+4\sin^2 x+3\sin 2x+6\cos x=0$

Bai 45: [ĐH A07] $\left(1+\sin^2 x\right)\cos x+\left(1+\cos^2 x\right)\sin x=1+\sin 2x$

Bai 46: [ĐH B07] $2\sin^2 2x+\sin 7x-1=\sin x$

Bai 47: [ĐH D07] $\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2+\sqrt{3}\cos x=2$

Bai 48: [Dự bị 1 ĐH A07] $\sin 2x+\sin x-\frac{1}{2\sin x}-\frac{1}{\sin 2x}=2\cot 2x$

Bai 49: [Dự bị 2 ĐH A07] $2\cos^2 x+2\sqrt{3}\sin x\cos x+1=3(\sin x+\sqrt{3}\cos x)$

Bai 50: [Dự bị 1 ĐH B07] $\sin\left(\frac{5x}{2}-\frac{\pi}{4}\right)-\cos\left(\frac{x}{2}-\frac{\pi}{4}\right)=\sqrt{2}\cos\frac{3x}{2}$

Bai 51: [Dự bị 2 ĐH B07] $\frac{\sin 2x}{\cos x}+\frac{\cos x}{\sin x}=\tan x-\cot x$

Bai 52: [Dự bị 1 ĐH D07] $2\sqrt{2}\sin\left(x-\frac{\pi}{12}\right)\cos x=1$

Bai 53: [Dự bị 2 ĐH D07] $(1-\tan x)(1+\sin 2x)=1+\tan x$

Bai 54: [ĐH A08] $\dfrac{1}{\sin x}+\dfrac{1}{\sin\left(x-\dfrac{3\pi}{2}\right)}=4\sin\left(\dfrac{7\pi}{4}-x\right)$

Bai 55: [ĐH B08] $\sin^3 x-\sqrt{3}\cos^3 x=\sin x\cos^2 x-\sqrt{3}\sin^2 x\cos x$

Bai 56: [ĐH D08] $2\sin x\left(1+\cos 2x\right)+\sin 2x=1+2\cos x$

Bai 57: [CĐ 08] $\sin 3x-\sqrt{3}\cos 3x=2\sin 2x$

Bai 58: [Dự bị 1 ĐH A08] $\tan x=\cot x+4\cos^2 2x$

Bai 59: [Dự bị 2 ĐH A08] $\sin\left(2x-\dfrac{\pi}{4}\right)=\sin\left(x-\dfrac{\pi}{4}\right)+\dfrac{\sqrt{2}}{2}$

Bai 60: [Dự bị 1 ĐH B08] $2\sin\left(x+\dfrac{\pi}{3}\right)-\sin\left(2x-\dfrac{\pi}{6}\right)=\dfrac{1}{2}$

Bai 61: [Dự bị 2 ĐH B08] $3\sin x+\cos 2x+\sin 2x=4\sin x\cos^2\dfrac{x}{2}$

Bai 62: [Dự bị 1 ĐH D08] $4\left(\sin^4 x+\cos^4 x\right)+\cos 4x+\sin 2x=0$

Bai 63: [Dự bị 2 ĐH D08] $\dfrac{\tan^2 x+\tan x}{\tan^2 x+1}=\dfrac{\sqrt{2}}{2}\sin\left(x+\dfrac{\pi}{4}\right)$

Bai 64: [ĐH A09] $\dfrac{(1-2\sin x)\cos x}{(1+2\sin x)(1-\sin x)}=\sqrt{3}$

Bai 65: [ĐH B09] $\sin x+\cos x\sin 2x+\sqrt{3}\cos 3x=2\left(\cos 4x+\sin^3 x\right)$

Bai 66: [ĐH D09] $\sqrt{3}\cos 5x-2\sin 3x\cos 2x-\sin x=0$

Bai 67: [CĐ 09] $(1+2\sin x)^2\cos x=1+\sin x+\cos x$

Bai 68: [ĐH A10] $\dfrac{\left(1+\sin x+\cos 2x\right)\sin\left(x+\dfrac{\pi}{4}\right)}{1+\tan x}=\dfrac{1}{\sqrt{2}}\cos x$

Bai 69: [ĐH B10] $\left(\sin 2x+\cos 2x\right)\cos x+2\cos 2x-\sin x=0$

Bai 70: [ĐH D10] $\sin 2x-\cos 2x+3\sin x-\cos x-1=0$

Bai 71: [ĐH A11] $\dfrac{1+\sin 2x+\cos 2x}{1+\cot^2 x}=\sqrt{2}\sin x\sin 2x$

Bai 72: [DB A11] $9\sin x+6\cos x-3\sin 2x+\cos 2x=8$

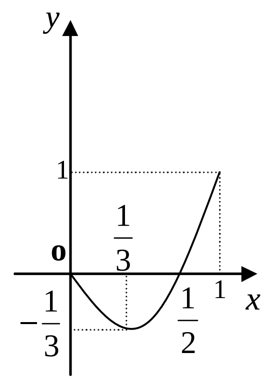
Bai 73: [ĐH B11] $\sin 2x\cos x+\sin x\cos x=\cos 2x+\sin x+\cos x$

Bai 74: [ĐH D11] $\dfrac{\sin 2x+2\cos x-\sin x-1}{\tan x+\sqrt{3}}=0$

Bai 75: [DB D11] $\sqrt{3}\cos 2x+2\cos x(\sin x-1)=0$

HƯỚNG DẪN GIẢI CÁC PHƯƠNG TRÌNH LƯỢNG GIÁC TRONG CÁC ĐỀ THI ĐẠI HỌC TỪ 2002 ĐẾN 2009

Bài	Hướng dẫn giải	Kết qủa
1 **A.2002**	Tìm $x\in(0;2\pi)$: $5\left(\sin x+\dfrac{\cos 3x+\sin 3x}{1+2\sin 2x}\right)=\cos 2x+3$ (1) Điều kiện : $\sin 2x\neq -\dfrac{1}{2}$ $5\left(\sin x+\dfrac{\cos 3x+\sin 3x}{1+2\sin 2x}\right)=5\left(\dfrac{\sin x+2\sin x\sin 2x+\cos 3x+\sin 3x}{1+\sin 2x}\right)$ $=5\left(\dfrac{\sin x+\cos x-\cos 3x+\cos 3x+\sin 3x}{1+2\sin 2x}\right)$ $=5\left(\dfrac{\sin 3x+\sin x+\cos x}{1+2\sin 2x}\right)=5\left(\dfrac{2\sin 2x\cos x+\cos x}{1+2\sin 2x}\right)$ $=5\left(\dfrac{\cos x(1+2\sin 2x)}{1+2\sin 2x}\right)=5\cos x$ $(1)\Leftrightarrow 5\cos x=\cos 2x+3\Leftrightarrow 2\cos^2 x-5\cos x+3=0$ $\Leftrightarrow\left[\begin{array}{l}\cos x=2\ \text{(L)}\\ \cos x=\dfrac{1}{2}=\cos\dfrac{\pi}{3}\end{array}\right.$	$\cos x=\cos\dfrac{\pi}{3}$ $\left[\begin{array}{l}x=\dfrac{\pi}{3}+k2\pi\\ x=-\dfrac{\pi}{3}+k2\pi\end{array}\right.$ $k\in\mathbb{Z}$ Vì $x\in(0;2\pi)$ Nên nghiệm của phương trình : $x=\dfrac{\pi}{3}$; $x=\dfrac{5\pi}{3}$
2 **B.2002**	$\sin^2 3x-\cos^2 4x=\sin^2 5x-\cos^2 6x$ $\Leftrightarrow\dfrac{1-\cos 6x}{2}-\dfrac{1+\cos 8x}{2}=\dfrac{1-\cos 10x}{2}-\dfrac{1+\cos 12x}{2}$	

	$\Leftrightarrow \cos 12x+\cos 10x=\cos 8x+\cos 6x$ $\Leftrightarrow 2\cos x(\cos 11x-\cos 7x)=0 \Leftrightarrow -4\cos x.\sin 9x.\sin 2x=0$	$\left[\begin{array}{l} x=\dfrac{k\pi}{9} \\ x=\dfrac{k\pi}{2} \end{array}\right. k\in\mathbb{Z}$
3 **D.2002**	Tìm $x\in[0;14]$: $\cos 3x-4\cos 2x+3\cos x-4=0$ (1) Ta có : $\cos 3x=4\cos^3 x-3\cos x$ $(1)\Leftrightarrow \cos 3x+3\cos x-4(1+\cos 2x)=0$ $\Leftrightarrow 4\cos^3 x-8\cos^2 x=0$ $\Leftrightarrow 4\cos^2 x\left(\cos x-2\right)=0 \Leftrightarrow \cos x=0$	$x=\dfrac{\pi}{2}+k\pi;k\in\mathbb{Z}$ Vì $x\in(0;14)$ $x\in\left\{\dfrac{\pi}{2};\dfrac{3\pi}{3};\dfrac{5\pi}{2};\dfrac{7\pi}{2}\right\}$
4 **DB 1** **2002**	Xác định m để phương trình sau có ít nhất 1 nghiệm thuộc $\left[0;\dfrac{\pi}{2}\right]$: $2\left(\sin^4 x+\cos^4 x\right)+\cos 4x+\sin 2x-m=0$ (1) $(1)\Leftrightarrow 2\left(1-2\sin^2 x\cos^2 x\right)+1-\sin^2 2x+2\sin 2x+m=0$ $\Leftrightarrow 3+m-3\sin^2 2x+2\sin 2x=0$ $\Leftrightarrow 3t^2-2t-(m+3)=0$ (2) với $t=\sin 2x$ Ta có : $x\in\left[0;\dfrac{\pi}{2}\right]\Leftrightarrow 2x\in[0;\pi]\Rightarrow t\in[0;1]$ *Bài toán thành* : Xác định m để phương trình sau có ít nhất 1 nghiệm thuộc đoạn $[0;1]$ $(2)\Leftrightarrow 3t^2-2t=m+3$ Đặt $\begin{cases} y=3t^2-2t & (P) \\ y=m+3 & d \end{cases}$ Số nghiệm của (2) là số giao điểm của d và (P)	

	Khảo sát hàm số : $y=3t^2-2t$ $t\in[0;1]$ $y'=6t-2$	Phương trình (2) có ít nhất một nghiện trên đoạn $[0;1]$

	$y'=0 \Leftrightarrow 6t-2=0 \Leftrightarrow t=\frac{1}{3}$ BBT $\begin{array}{c\|ccccc} x & -\infty \quad 0 & & \frac{1}{3} & & 1 \quad +\infty \\ \hline y' & \blacksquare & - & 0 & + & \blacksquare \\ \hline y & \blacksquare & 0 \searrow & -\frac{1}{3} & \nearrow & \blacksquare \end{array}$	$\Leftrightarrow -\frac{1}{3} \le m+3 \le 1$ $\Leftrightarrow -\frac{10}{3} \le m \le -2$
5 **DB 2** **2002**	$\frac{\sin^4 x+\cos^4 x}{5\sin 2x}=\frac{1}{2}\cot 2x-\frac{1}{8\sin 2x} \quad (1)$ Điều kiện : $\sin 2x \ne 0$ $(1) \Leftrightarrow \frac{1-2\sin^2 x\cos^2 x}{5}=\frac{1}{2}\cos 2x-\frac{1}{8}$ $\Leftrightarrow 1-\frac{\sin^2 2x}{2}=\frac{5}{2}\cos 2x-\frac{5}{8} \Leftrightarrow 2-(1-\cos^2 2x)=5\cos 2x-\frac{5}{4}$ $\Leftrightarrow \cos^2 2x-5\cos 2x+\frac{9}{4}=0 \Leftrightarrow \left[\begin{array}{l}\cos 2x=\frac{9}{2} (L)\\ \cos 2x=\frac{1}{2}\end{array}\right.$	$\cos 2x=\cos\frac{\pi}{3}$ $\Leftrightarrow \left[\begin{array}{l}2x=\frac{\pi}{3}+k2\pi\\ x=-\frac{\pi}{3}+k2\pi\end{array}\right.$ $\Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{6}+k\pi\\ x=-\frac{\pi}{6}+k\pi\end{array}\right. \quad k\in\mathbb{Z}$
6 **DB 3** **2002**	$\tan^4 x+1=\frac{\left(2-\sin^2 2x\right)\sin 3x}{\cos^4 x} \quad (1)$ Điều kiện : $\cos x \ne 0$ $(1) \Leftrightarrow \sin^4 x+\cos^4 x=(2-\sin^2 2x)\sin 3x$ $\Leftrightarrow 1-\frac{\sin^2 2x}{2}=(2-\sin^2 2x)\sin 3x$ $\Leftrightarrow 2-\sin^2 2x=(2-\sin^2 2x)2\sin 3x$ $\Leftrightarrow (2-\sin^2 2x)(1-2\sin 3x)=0$ $\Leftrightarrow 1-2\sin 3x=0 \Leftrightarrow \sin 3x=\frac{1}{2}$	$\sin 3x=\sin\frac{\pi}{6}$ $\left[\begin{array}{l}3x=\frac{\pi}{6}+k2\pi\\ 3x=\frac{5\pi}{6}+k2\pi\end{array}\right.$ $\left[\begin{array}{l}x=\frac{\pi}{18}+\frac{k2\pi}{3}\\ x=\frac{5\pi}{18}+\frac{k2\pi}{3}\end{array}\right.$ $k\in\mathbb{Z}$

7 **DB 4** **2002**	$\tan x+\cos x-\cos^2 x=\sin x\left(1+\tan x.\tan\frac{x}{2}\right)$ (1) Điều kiện : $\begin{cases}\cos x\neq 0\\ \cos\frac{x}{2}\neq 0\end{cases}$ Ta có : $1+\tan x.\tan\frac{x}{2}=1+\dfrac{\sin x\sin\frac{x}{2}}{\cos x\cos\frac{x}{2}}=\dfrac{\cos x\cos\frac{x}{2}+\sin x\sin\frac{x}{2}}{\cos x\cos\frac{x}{2}}$ $=\dfrac{\cos\left(x-\frac{x}{2}\right)}{\cos x\cos\frac{x}{2}}=\dfrac{1}{\cos x}$	
	$(1)\Leftrightarrow \tan x+\cos x-\cos^2 x=\dfrac{\sin x}{\cos x}$ $\Leftrightarrow \cos x(1-\cos x)=0\Leftrightarrow \left[\begin{array}{l}\cos x=0\ \ (L)\\ \cos x=1\end{array}\right.$	$\cos x=1$ $\Leftrightarrow x=k2\pi;k\in\mathbb{Z}$
8 **DB 5** **2002**	Cho phương trình : $\dfrac{2\sin x+\cos x+1}{\sin x-2\cos x+3}=a$ a) Giải phương trình với $a=\frac{1}{3}$ b) Tìm a để phương trình trên có nghiệm. ***Giải.*** a)Với $a=\frac{1}{3}$, phương trình thành : $\dfrac{2\sin x+\cos x+1}{\sin x-2\cos x+3}=\dfrac{1}{3}$ (1) vì : $\sin x-2\cos x+3>0\ \ \forall x\in\mathbb{R}$ $\Leftrightarrow 6\sin x+3\cos x+3=\sin x-2\cos x+3$ (1) $\Leftrightarrow 5\sin x+5\cos x=0\Leftrightarrow \sin x+\cos x=0$ $\Leftrightarrow \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=0\Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=0$	$\sin\left(x+\frac{\pi}{4}\right)=0$ $\Leftrightarrow x+\frac{\pi}{4}=k\pi$ $\Leftrightarrow x=-\frac{\pi}{4}+k\pi$ $k\in\mathbb{Z}$
	b) $\dfrac{2\sin x+\cos x+1}{\sin x-2\cos x+3}=a\Leftrightarrow \sin x+\cos x+1=a(\sin x-2\cos x+3)$ $\Leftrightarrow (2-a)\sin x+(2a+1)\cos x=3a-1$ (2)	$-\frac{1}{2}\le a\le 2$

	Điều kiện để phương trình (2) có nghiệm : $\left(2-a\right)^2+\left(2a+1\right)^2\geq\left(3a-1\right)^2\Leftrightarrow 4a^2-6a-4\leq 0$ $\Leftrightarrow -\frac{1}{2}\leq a\leq 2$	
9 **DB 6** **2002**	$\sqrt{\frac{1}{8\cos^2 x}}=\sin x \qquad (1)$ Điều kiện : $\begin{cases}\cos x\neq 0\\ \sin x\geq 0\end{cases}$ $(1)\Leftrightarrow\frac{1}{8\cos^2 x}=\sin^2 x\Leftrightarrow 1=8\sin^2 x\cos^2 x$ $\Leftrightarrow 2\sin^2 2x-1=0\Leftrightarrow\cos 4x=0\Leftrightarrow 4x=\frac{\pi}{2}+k\pi\Leftrightarrow x=\frac{\pi}{8}+\frac{k\pi}{4}$	Vì : $\sin x\geq 0$ $x=\frac{\pi}{8}+m2\pi$ $x=\frac{3\pi}{8}+m2\pi\ ;m\in\mathbb{Z}$ $x=\frac{5\pi}{8}+m2\pi$ $x=\frac{7\pi}{8}+m2\pi$
10 **A2003**	$\cot x-1=\frac{\cos 2x}{1+\tan x}+\sin^2 x-\frac{1}{2}\sin 2x \quad (1)$ Điều kiện : $\begin{cases}\sin 2x\neq 0\\ \tan x\neq -1\end{cases}$ $(1)\Leftrightarrow\frac{\cos x}{\sin x}-1=\frac{\cos^2 x-\sin^2 x}{1+\frac{\sin x}{\cos x}}+\sin x(\sin x-\cos x)$ $\Leftrightarrow\frac{\cos x-\sin x}{\sin x}=\frac{\cos x(\cos^2 x-\sin^2 x)}{\sin x+\cos x}+\sin x(\sin x-\cos x)$ $\Leftrightarrow\frac{\cos x-\sin x}{\sin x}=\cos x(\cos x-\sin x)+\sin x(\sin x-\cos x)$ $\Leftrightarrow(\cos x-\sin x)\left(\sin^2 x-\sin x\cos x+1\right)=0$ $\Leftrightarrow\left[\begin{array}{l}\cos x-\sin x=0\\ \sin^2 x-\sin x\cos x+1=0\end{array}\right.$	
	* $\cos x-\sin x=0\Leftrightarrow\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)=0$ $\Leftrightarrow\cos\left(x+\frac{\pi}{4}\right)=0\Leftrightarrow x+\frac{\pi}{4}=\frac{\pi}{2}+k\pi\Leftrightarrow x=\frac{\pi}{4}+k\pi;k\in\mathbb{Z}$	$x=\frac{\pi}{4}+k\pi;k\in\mathbb{Z}$

	* $\sin^2 x - \sin x\cos x + 1 = 0 \Leftrightarrow \frac{1-\cos 2x}{2} - \frac{\sin 2x}{2} + 1 = 0$ $\Leftrightarrow \sin 2x + \cos 2x - 3 = 0$ (vô nghiệm)	
11 **B2003**	$\cot x - \tan x + 4\sin 2x = \frac{2}{\sin 2x}$ (1) Điều kiện : $\sin 2x \neq 0$ $(1) \Leftrightarrow \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} + 4\sin 2x = \frac{2}{\sin 2x}$ $\Leftrightarrow \frac{\cos^2 x - \sin^2 x}{\sin x\cos x} + 4\sin 2x = \frac{2}{\sin 2x}$ $\Leftrightarrow 2\cos 2x + 4\sin^2 2x = 2 \Leftrightarrow 2\cos 2x + 4\left(1-\cos^2 2x\right) = 2$ $\Leftrightarrow 2\cos^2 x - \cos 2x - 1 = 0 \Leftrightarrow \left[\begin{array}{l} \cos 2x = 1 \\ \cos 2x = -\frac{1}{2} \end{array}\right.$	$\left[\begin{array}{l} x = k\pi \\ x = \pm\frac{\pi}{3} + k\pi \end{array}\right. k \in \mathbb{Z}$
12 **D2003**	$\sin^2\left(\frac{x}{2} - \frac{\pi}{4}\right)\tan^2 x - \cos^2\frac{x}{2} = 0$ (1) Điều kiện : $\cos x \neq 0$ $(1) \Leftrightarrow \frac{1}{2}\left[1 - \cos\left(x - \frac{\pi}{2}\right)\right]\frac{\sin^2 x}{\cos^2 x} = \frac{1}{2}(1+\cos x)$ $\Leftrightarrow (1-\sin x)\sin^2 x = (1+\cos x)\cos^2 x$ $\Leftrightarrow (1-\sin x)\left(1-\cos^2 x\right) = (1+\cos x)\left(1-\sin^2 x\right)$ $\Leftrightarrow (1-\sin x)(1+\cos x)(\sin x + \cos x) = 0$ $\Leftrightarrow \left[\begin{array}{l} \sin x = 1 \\ \cos x = -1 \\ \sin x + \cos x = 0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \sin x = 1 \\ \cos x = -1 \\ \sin\left(x + \frac{\pi}{4}\right) = 0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{2} + k\pi \\ x = \pi + k2\pi \\ x = -\frac{\pi}{4} + k\pi \end{array}\right.$	So với điều kiện : $\cos x \neq 0$ Nghiệm của (1) : $\left[\begin{array}{l} x = \pi + k2\pi \\ x = -\frac{\pi}{4} + k\pi \end{array}\right. k \in \mathbb{Z}$

13 **DB 1** **A2003**	$3-\tan x(\tan x+2\sin x)+6\cos x=0$ Điều kiện : $\cos x\neq 0$ $\Leftrightarrow 3-\frac{\sin x}{\cos x}\left(\frac{\sin x+2\sin x\cos x}{\cos x}\right)+6\cos x=0$ $\Leftrightarrow 3\cos^2 x-\sin^2 x(1+2\cos x)+6\cos^2 x=0$ $\Leftrightarrow 3\cos^2 x(1+2\cos x)-\sin^2 x(1+2\cos x)=0$ $\Leftrightarrow (1+2\cos x)(3\cos^2 x-\sin^2 x)=0$ $\Leftrightarrow \left[\begin{array}{l}1+2\cos x=0\\4\cos^2 x-1=0\end{array}\right.\Leftrightarrow\left[\begin{array}{l}\cos x=-\frac{1}{2}\\\cos^2 x=\frac{1}{4}\end{array}\right.\Leftrightarrow \cos^2 x=\frac{1}{4}$ $\Leftrightarrow 1+\cos x=\frac{1}{2}\Leftrightarrow \cos 2x=-\frac{1}{2}=\cos\frac{2\pi}{3}$	$\cos 2x=\cos\frac{2\pi}{3}$ $\Leftrightarrow\left[\begin{array}{l}2x=\frac{2\pi}{3}+k2\pi\\2x=-\frac{2\pi}{3}+k2\pi\end{array}\right.$ $\Leftrightarrow\left[\begin{array}{l}x=\frac{\pi}{3}+k\pi\\x=-\frac{\pi}{3}+k\pi\end{array}\right.\quad k\in\mathbb{Z}$
14 **DB 2** **A2003**	$\cos 2x+\cos x(2\tan^2 x-1)=2$ Điều kiện : $\cos x\neq 0$ $\Leftrightarrow \cos 2x+\frac{2\sin^2 x}{\cos x}-\cos x=2$ $\Leftrightarrow \frac{2\sin^2 x}{\cos x}-\cos x=2-\cos 2x=1+2\sin^2 x$ $\Leftrightarrow 2\sin^2 x\left(\frac{1}{\cos x}-1\right)=1+\cos x$ $\Leftrightarrow 2(1-\cos^2 x)(1-\cos x)=(1+\cos x)\cos x$ $\Leftrightarrow (1+\cos x)\left[2(1-\cos x)^2-\cos x\right]=0$ $\Leftrightarrow\left[\begin{array}{l}\cos x=-1\\2\cos^2 x-5\cos x+2=0\end{array}\right.\Leftrightarrow\left[\begin{array}{l}\cos x=-1\\\cos x=\frac{1}{2}\end{array}\right.$	$\left[\begin{array}{l}x=\pi+k\pi\\x=\pm\frac{\pi}{3}+k2\pi\end{array}\right.$
15 **DB 1** **B2003**	$3\cos 4x-8\cos^6 x+2\cos^2 x+3=0$ $\Leftrightarrow 3(1+\cos 4x)-2\cos^2 x(4\cos^4 x-1)=0$	$\left[\begin{array}{l}x=\frac{\pi}{4}+\frac{k\pi}{2}\\x=k\pi\end{array}\right.\quad k\in\mathbb{Z}$

	$\Leftrightarrow 6\cos^2 2x - 2\cos^2 x(2\cos^2 x - 1)(2\cos^2 x + 1) = 0$ $\Leftrightarrow 6\cos^2 2x - \cos^2 x(2\cos^2 x + 1)\cos 2x = 0$ $\Leftrightarrow \cos 2x\left[3\cos 2x - \cos^2 x(2\cos^2 x + 1)\right] = 0$ $\Leftrightarrow \cos 2x\left(2\cos^4 x - 5\cos^2 x + 3\right) = 0 \Leftrightarrow \left[\begin{array}{l}\cos 2x = 0\\ 2\cos^4 x - 5\cos^2 x + 3 = 0\end{array}\right.$ * $\cos 2x = 0 \Leftrightarrow 2x = \frac{\pi}{2} + k\pi \Leftrightarrow x = \frac{\pi}{4} + \frac{k\pi}{2}$; $k \in \mathbb{Z}$ * $2\cos^4 x - 5\cos^2 x + 3 = 0 \Leftrightarrow \left[\begin{array}{l}\cos^2 x = 1\\ \cos^2 x = \frac{3}{2} (L)\end{array}\right. \Leftrightarrow \sin^2 x = 0$	
16 **DB 2** **B2003**	$\frac{\left(2-\sqrt{3}\right)\cos x - 2\sin^2\left(\frac{x}{2} - \frac{\pi}{4}\right)}{2\cos x - 1} = 1 \quad (1)$ Điều kiện : $\cos x \neq \frac{1}{2}$ $\Leftrightarrow (2-\sqrt{3})\cos x - \left[1 - \cos\left(x - \frac{\pi}{2}\right)\right] = 2\cos x - 1$ $\Leftrightarrow 2\cos x - \sqrt{3}\cos x - 1 + \sin x = 2\cos x - 1$ (1) $\Leftrightarrow \sqrt{3}\cos x - \sin x = 0$ $\Leftrightarrow \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = 0 \Leftrightarrow \cos x\cos\frac{\pi}{6} - \sin x\sin\frac{\pi}{6} = 0$ $\Leftrightarrow \cos\left(x + \frac{\pi}{6}\right) = 0 \Leftrightarrow x + \frac{\pi}{6} = \frac{\pi}{2} + k\pi \Leftrightarrow x = \frac{\pi}{3} + k\pi; k \in \mathbb{Z}$	Vì : $\cos x \neq \frac{1}{2}$ Nên nghiệm của phương trình : $x = \frac{4\pi}{3} + k2\pi; k \in \mathbb{Z}$
17 **DB 1** **D2003**	$\frac{\cos^2 x\left(\cos x - 1\right)}{\sin x + \cos x} = 2\left(1 + \sin x\right) \quad (1)$ Điều kiện : $\sin x + \cos x = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right) \neq 0$ (1) $\Leftrightarrow (1 - \sin^2 x)(\cos x - 1) = 2(1 + \sin x)(\sin x + \cos x)$	$\left[\begin{array}{l}x = -\frac{\pi}{2} + k2\pi\\ x = \pi + k2\pi\end{array}\right. k \in \mathbb{Z}$

	$\Leftrightarrow (1+\sin x)\left[(1-\sin x)(\cos x-1)-2(\sin x+\cos x)\right]=0$ $\Leftrightarrow (1+\sin x)\left[\cos x-1-\sin x\cos x+\sin x-2\sin x-2\cos x\right]=0$ $\Leftrightarrow (1+\sin x)\left[\sin x+1+\sin x\cos x+\cos x\right]=0$ $\Leftrightarrow (1+\sin x)\left[(1+\sin x)+\cos x(1+\sin x)\right]=0$ $\Leftrightarrow (1+\sin x)^2(1+\cos x)=0 \Leftrightarrow \left[\begin{array}{l}\sin x=-1\\ \cos x=-1\end{array}\right.$	
18 **DB 2** **D2003**	$\cot x = \tan x + \dfrac{2\cos 4x}{\sin 2x}$ (1) Điều kiện : $\sin 2x \neq 0 \Leftrightarrow \cos 2x \neq \pm 1$ $(1) \Leftrightarrow \cot x-\tan x=\dfrac{2\cos 4x}{\sin 2x}$ $\Leftrightarrow \dfrac{\cos x}{\sin x}-\dfrac{\sin x}{\cos x}=\dfrac{\cos 4x}{\sin x\cos x} \Leftrightarrow \cos^2 x-\sin^2 x=\cos 4x$ $\Leftrightarrow \cos 2x=\cos 4x \Leftrightarrow 2\cos^2 2x-\cos 2x-1=0$ $\Leftrightarrow \left[\begin{array}{l}\cos 2x=1 (L)\\ \cos 2x=-\dfrac{1}{2}=\cos\dfrac{2\pi}{3}\end{array}\right.$	$x=\pm\dfrac{\pi}{3}+k\pi; k\in\mathbb{Z}$
19 **B2004**	$5\sin x-2=3(1-\sin x)\tan^2 x$ Điều kiện : $\cos x\neq 0$ $\Leftrightarrow 5\sin x-2=\dfrac{3\sin^2 x}{1-\sin^2 x}(1-\sin x)$ $\Leftrightarrow (5\sin x-2)(1+\sin x)=3\sin^2 x$ $\Leftrightarrow 2\sin^2 x+3\sin x-2=0 \Leftrightarrow \left[\begin{array}{l}\sin x=\dfrac{1}{2}=\sin\dfrac{\pi}{6}\\ \sin x=-2\end{array}\right.$	$\left[\begin{array}{l}x=\dfrac{\pi}{6}+k2\pi\\ x=\dfrac{5\pi}{6}+k2\pi\end{array}\right. k\in\mathbb{Z}$
20 **D2004**	$(2\cos x-1)(2\sin x+\cos x)=\sin 2x-\sin x$ $\Leftrightarrow (2\cos x-1)(2\sin x+\cos x)=\sin x(2\cos x-1)$ $\Leftrightarrow (2\cos x-1)(\sin x+\cos x)=0$ $\Leftrightarrow \left[\begin{array}{l}2\cos x=1\\ \sin x+\cos x=0\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\cos x=\dfrac{1}{2}\\ \sqrt{2}\sin\left(x+\dfrac{\pi}{4}\right)=0\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\cos x=\cos\dfrac{\pi}{3}\\ \sin\left(x+\dfrac{\pi}{4}\right)=0\end{array}\right.$	$\left[\begin{array}{l}x=\pm\dfrac{\pi}{3}+k2\pi\\ x=-\dfrac{\pi}{4}+k\pi\end{array}\right. k\in\mathbb{Z}$

21 **DB 1** **A2004**	$\sin x+\sin 2x=\sqrt{3}\left(\cos x+\operatorname{cox}2x\right)$ $\Leftrightarrow \sin x+\sin 2x=\sqrt{3}\cos x+\sqrt{3}\cos 2x$ $\Leftrightarrow \sin x-\sqrt{3}\cos x=\sqrt{3}\cos 2x-\sin 2x$ $\Leftrightarrow \frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\cos x=\frac{\sqrt{3}}{2}\cos 2x-\frac{1}{2}\sin 2x$	
	$\Leftrightarrow \sin\left(x-\frac{\pi}{3}\right)=\cos\left(2x+\frac{\pi}{6}\right)$ $\Leftrightarrow \sin\left(x-\frac{\pi}{3}\right)=\cos\left[\frac{\pi}{2}+\left(2x-\frac{\pi}{3}\right)\right]=-\sin\left(2x-\frac{\pi}{3}\right)$ $\Leftrightarrow \sin\left(2x-\frac{\pi}{3}\right)+\sin\left(x-\frac{\pi}{3}\right)=0$ $\Leftrightarrow 2\sin\left(\frac{3x}{2}-\frac{\pi}{3}\right)\cos\left(\frac{x}{2}\right)=0\Leftrightarrow \left[\begin{array}{l}\sin\left(\frac{3x}{2}-\frac{\pi}{3}\right)=0\\ \cos\frac{x}{2}=0\end{array}\right.$	$\left[\begin{array}{l}\frac{3x}{2}-\frac{\pi}{3}=k\pi\\ \frac{x}{2}=\frac{\pi}{2}+k\pi\end{array}\right.$ $\Leftrightarrow \left[\begin{array}{l}x=\frac{2\pi}{9}+\frac{k2\pi}{3}\\ x=\pi+k2\pi\end{array}\right.$ $k\in\mathbb{Z}$
22 **DB 2** **A2004**	$\sqrt{1-\sin x}+\sqrt{1-\cos x}=1\quad (1)\qquad$ TXĐ : $D=\mathbb{R}$ ***Chú ý*** : $1-\sin x\ge 0\ ;\ 1-\cos x\ge 0$ $(1)\Leftrightarrow 2-(\sin x+\cos x)+2\sqrt{(1-\sin x)(1-\cos x)}=1$ $\Leftrightarrow 2-(\sin x+\cos x)+2\sqrt{1-(\sin x+\cos x)-\sin x\cos x}=1\quad (2)$ Đặt : $t=\sin x+\cos x\ ;\ \lvert t\rvert\le\sqrt{2}$,khi đó : $\sin x\cos x=\frac{t^2-1}{2}$ $(2)\Leftrightarrow 1-t+2\sqrt{\frac{t^2-2t+1}{2}}=0$ $\Leftrightarrow 1-t+\sqrt{2}\sqrt{(t-1)^2}=0\Leftrightarrow 1-t+\sqrt{2}\lvert t-1\rvert=0$ $\Leftrightarrow \sqrt{2}\lvert t-1\rvert=t-1\quad (3)$ (nhận xét và suy ra : $t\ge 1$) $(3)\ \Leftrightarrow t=1\Leftrightarrow \sin x+\cos x=1\Leftrightarrow \cos\left(x-\frac{\pi}{4}\right)=\cos\frac{\pi}{4}$	$\Leftrightarrow \left[\begin{array}{l}x+\frac{\pi}{4}=\frac{\pi}{4}+k2\pi\\ x+\frac{\pi}{4}=-\frac{\pi}{4}+k2\pi\end{array}\right.$ $\Leftrightarrow \left[\begin{array}{l}\frac{\pi}{2}+k2\pi\\ x=k2\pi\end{array}\right.\ k\in\mathbb{Z}$ $k\in\mathbb{Z}$

23 **DB 1** **B2004**	$4\left(\sin^3 x+\cos^3 x\right)=\cos x+3\sin x$ $\Leftrightarrow 4\sin^3 x+4\cos^3 x-\cos x-3\sin x=0$ $\Leftrightarrow 4\sin^3 x+4\cos x(1-\sin^2 x)-\cos x-3\sin x=0$ $\Leftrightarrow 4\sin^3 x+3\cos x-4\sin^2 x\cos x-3\sin x=0$ $\Leftrightarrow 3(\cos x-\sin x)-4\sin^2 x(\cos x-\sin x)=0$ $\Leftrightarrow (\cos x-\sin x)\left(3-4\sin^2 x\right)=0$ $\Leftrightarrow \left[\begin{array}{l}\cos x-\sin x=0\\ \sin^2 x=\dfrac{3}{4}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\sqrt{2}\cos\left(x-\dfrac{\pi}{4}\right)=0\\ \sin x=\dfrac{\sqrt{3}}{2}\\ \sin x=-\dfrac{\sqrt{3}}{2}\end{array}\right.$	$\left[\begin{array}{l}x=\dfrac{\pi}{4}+k\pi\\ x=\pm\dfrac{\pi}{3}+k\pi\end{array}\right. \quad k\in\mathbb{Z}$
24 **DB 2** **B2004**	$\dfrac{1}{\cos x}-\dfrac{1}{\sin x}=2\sqrt{2}\cos\left(x+\dfrac{\pi}{4}\right)$ Điều kiện : $\sin 2x\neq 0$ (1) $\Leftrightarrow \sin x-\cos x=2\sqrt{2}\cos\left(x+\dfrac{\pi}{4}\right)$	
	$\Leftrightarrow -\sqrt{2}\cos\left(x+\dfrac{\pi}{4}\right)=2\sqrt{2}\cos\left(x+\dfrac{\pi}{4}\right).\dfrac{1}{2}\sin 2x$ $\Leftrightarrow \cos\left(x+\dfrac{\pi}{4}\right)\left(1+\sin 2x\right)=0$ $\Leftrightarrow \left[\begin{array}{l}\cos\left(x+\dfrac{\pi}{4}\right)=0\\ \sin 2x=-1\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x+\dfrac{\pi}{4}=\dfrac{\pi}{2}+k\pi\\ 2x=-\dfrac{\pi}{2}+k2\pi\end{array}\right. \quad k\in\mathbb{Z}$	$\left[\begin{array}{l}x=\dfrac{\pi}{4}+k\pi\\ x=-\dfrac{\pi}{4}+k\pi\end{array}\right. \quad k\in\mathbb{Z}$ $\Leftrightarrow x=\dfrac{\pi}{4}+\dfrac{k\pi}{2}$
25 **DB 1** **D2004**	$\sin 4x\sin 7x=\cos 3x\cos 6x$ $\Leftrightarrow -\dfrac{1}{2}\left(\cos 11x-\cos(-3x)\right)=\dfrac{1}{2}\left(\cos 9x+\cos 3x\right)$ $\Leftrightarrow -\cos 11x+\cos 3x=\cos 9x+\cos 3x$ $\Leftrightarrow \cos 11x+\cos 9x=0$ $\Leftrightarrow 2\cos 10x\cos x=0 \Leftrightarrow \left[\begin{array}{l}\cos 10x=0\\ \cos x=0\end{array}\right. \Leftrightarrow \left[\begin{array}{l}10x=\dfrac{\pi}{2}+k\pi\\ x=\dfrac{\pi}{2}+k\pi\end{array}\right.$	$\left[\begin{array}{l}x=\dfrac{\pi}{20}+k10\pi\\ x=\dfrac{\pi}{2}+k\pi\end{array}\right. \quad k\in\mathbb{Z}$

26 **DB 2** **D2004**	$\sin 2x-2\sqrt{2}\left(\sin x+\cos x\right)-5=0 \quad (1)$ Đặt $t=\sin x+\cos x$ với $-\sqrt{2}\le t\le\sqrt{2}$ $\Rightarrow \sin 2x=t^2-1$ (1) $\Leftrightarrow t^2-2\sqrt{2}t-6=0 \Leftrightarrow \left[\begin{array}{l} t=3\sqrt{2} \\ t=-\sqrt{2} \end{array}\right.$ Với $t=-\sqrt{2} \Leftrightarrow \sin x+\cos x=-\sqrt{2}$ $\Leftrightarrow \cos\left(x-\frac{\pi}{4}\right)=-1 \Leftrightarrow x-\frac{\pi}{4}=\pi+k2\pi \quad k\in\mathbb{Z}$	$x=\frac{5\pi}{4}+k2\pi; k\in\mathbb{Z}$
27 **A2005**	$\cos^2 3x\cos 2x-\cos^2 x=0$ $\Leftrightarrow \frac{(1+\cos 6x)\cos 2x}{2}-\frac{1+\cos 2x}{2}=0$ $\Leftrightarrow \cos 2x+\cos 6x\cos 2x-1-\cos 2x=0$ $\Leftrightarrow \cos 6x\cos 2x-1=0 \Leftrightarrow \cos 8x+\cos 4x-2=0$ $\Leftrightarrow 2\cos^2 4x-1+\cos 4x-2=0$ $\Leftrightarrow 2\cos^2 4x+\cos 4x-3=0 \Leftrightarrow \left[\begin{array}{l} \cos 4x=1 \\ \cos 4x=-\frac{3}{2} \end{array}\right.$	$x=\frac{k\pi}{2} \quad ,k\in\mathbb{Z}$
28 **B2005**	$1+\sin+\cos x+\sin 2x+\cos 2x=0$ $\Leftrightarrow \sin x+\cos x+2\sin x\cos x+2\cos^2 x=0$ $\Leftrightarrow (\sin x+\cos x)+2\cos x(\sin x+\cos x)=0$ $\Leftrightarrow (\sin x+\cos x)(1+2\cos x)=0 \Leftrightarrow \left[\begin{array}{l} \sin x+\cos x=0 \\ \cos x=-\frac{1}{2}=\cos\frac{2\pi}{3} \end{array}\right.$	$\left[\begin{array}{l} x=-\frac{\pi}{4}+k\pi \\ x=\pm\frac{2\pi}{3}+k2\pi \end{array}\right. \quad k\in\mathbb{Z}$
29 **D2005**	$\cos^4 x+\sin^4 x+\cos\left(x-\frac{\pi}{4}\right)\sin\left(3x-\frac{\pi}{4}\right)-\frac{3}{2}=0$ $\Leftrightarrow 1-2\sin^2 x\cos^2 x+\frac{1}{2}\left[\sin\left(4x-\frac{\pi}{2}\right)+\sin 2x\right]-\frac{3}{2}=0$ $\Leftrightarrow 2-\sin^2 2x-\cos 4x+\sin 2x-3=0$ $\Leftrightarrow -\sin^2 2x-(1-2\sin^2 2x)+\sin 2x-1=0$ $\Leftrightarrow \sin^2 2x+\sin 2x-2=0 \Leftrightarrow \left[\begin{array}{l} \sin 2x=1 \\ \sin 2x=-2 \end{array}\right.$	$x=\frac{\pi}{4}+k\pi \; ; \; k\in\mathbb{Z}$

30 **DB 1** **A2005**	Tìm $x\in(0;\pi)$ của : $4\sin^2\frac{x}{2}-\sqrt{3}\cos 2x=1+2\cos^2\left(x-\frac{3\pi}{4}\right)$ $\Leftrightarrow 2(1-\cos x)-\sqrt{3}\cos 2x=1+1+\cos\left(2x-\frac{3\pi}{2}\right)$ $\Leftrightarrow 2-2\cos x-\sqrt{3}\cos 2x=2-\sin 2x$ $\Leftrightarrow -2\cos x=\sqrt{3}\cos 2x-\sin 2x$ (chia 2 vế cho 2) $\Leftrightarrow -\cos x=\frac{\sqrt{3}}{2}\cos 2x-\frac{1}{2}\sin 2x\Leftrightarrow \cos(\pi-x)=\cos\left(2x+\frac{\pi}{6}\right)$ $\Leftrightarrow \cos\left(2x+\frac{\pi}{6}\right)=\cos(\pi-x)\Leftrightarrow\left[\begin{array}{l}2x+\frac{\pi}{6}=\pi-x+k2\pi\\2x+\frac{\pi}{6}=-\pi+x+k2\pi\end{array}\right.$ $\Leftrightarrow\left[\begin{array}{l}x=\frac{5\pi}{18}+\frac{k_1 2\pi}{3}\\x=-\frac{7\pi}{6}+k_2 2\pi\end{array}\right.\quad k_1;k_2\in\mathbb{Z}$ Vì $\begin{cases}k_1\in\mathbb{Z}\\k_1\in(0;\pi)\end{cases}\Rightarrow k_1\in\{0;1\}\Rightarrow x=\frac{5\pi}{18};x=\frac{17\pi}{18}$ Vì $\begin{cases}k_2\in\mathbb{Z}\\k_2\in(0;\pi)\end{cases}\Rightarrow k_2=1\Rightarrow x=\frac{5\pi}{6}$	$\cos\left(2x-\frac{3\pi}{2}\right)$ $=-\sin 2x$ $\Leftrightarrow\left[\begin{array}{l}x=\frac{5\pi}{18}\\x=\frac{17\pi}{18}\\x=\frac{5\pi}{6}\end{array}\right.$
31 **DB 2** **A2005** **32** **DB1** **B2005**	$2\sqrt{2}\cos^3\left(x-\frac{\pi}{4}\right)-3\cos x-\sin x=0$ $\Leftrightarrow\left[\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)\right]^3-3\cos x-\sin x=0$ $\Leftrightarrow(\cos x+\sin x)^3-3\cos x-\sin x=0$ $\Leftrightarrow\cos^3 x+\sin^3 x+3\cos^2 x\sin x+3\cos x\sin^2 x-3\cos x-\sin x=0$ $\Leftrightarrow\left[\begin{array}{l}\begin{cases}\cos x=0\\\sin^3 x-\sin x=0\end{cases}\\\begin{cases}\cos x\neq 0\\1+\tan^3 x+3\tan x+3\tan^2 x-3(1+\tan^2 x)-\tan x(1+\tan^2 x)=0\end{cases}\end{array}\right.$	$\left[\begin{array}{l}x=\frac{\pi}{2}+k\pi\\x=\frac{\pi}{4}+k\pi\end{array}\right.\quad k\in\mathbb{Z}$

	$\Leftrightarrow \left[\begin{array}{l}\sin^2 x = 1 \\ \tan x = 1\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\cos^2 x = 0 \\ \tan x = 1\end{array}\right.$	
33 **DB 2** **B2005**	$\tan\left(\dfrac{\pi}{2} + x\right) - 3\tan^2 x = \dfrac{\cos 2x - 1}{\cos^2 x}$ (1) Điều kiện : $\sin 2x \neq 0$ $(1) \Leftrightarrow -\cot x - 3\tan^2 x = -\dfrac{2\sin^2 x}{\cos^2 x}$ $\Leftrightarrow \dfrac{1}{\tan x} + \tan^2 x = 0 \Leftrightarrow \tan^3 x = -1 \Leftrightarrow \tan x = -1$	$x = -\dfrac{\pi}{4} + k\pi\ ;\ k \in \mathbb{Z}$
34 **DB 1** **D2005**	$\tan\left(\dfrac{3\pi}{2} - x\right) + \dfrac{\sin x}{1 + \cos x} = 2$ (1) Điều kiện : $\sin x \neq 0$ $\Leftrightarrow \cot x + \dfrac{\sin x}{1 + \cos x} = 2 \Leftrightarrow \dfrac{\cos x}{\sin x} + \dfrac{\sin x}{1 + \cos x} = 2$ $\Leftrightarrow \cos x(1 + \cos x) + \sin^2 x = 2\sin x(1 + \cos x)$ (1) $\Leftrightarrow \cos x + \cos^2 x + \sin^2 x = 2\sin x(1 + \cos x)$ $\Leftrightarrow (1 + \cos x)(1 - 2\sin x) = 0 \Leftrightarrow \left[\begin{array}{l}\cos x = -1 (L) \\ \sin x = \dfrac{1}{2} = \sin\dfrac{\pi}{6}\end{array}\right.$	$\left[\begin{array}{l}x = \dfrac{\pi}{6} + k2\pi \\ x = \dfrac{5\pi}{6} + k2\pi\end{array}\right.\ k \in \mathbb{Z}$
35 **DB 2** **D2005**	$\sin 2x + \cos 2x + 3\sin x - \cos x - 2 = 0$ $\Leftrightarrow 2\sin x\cos x + 1 - 2\sin^2 x + 3\sin x - \cos x - 2 = 0$ $\Leftrightarrow 2\sin^2 x - (2\cos x + 3)\sin x + \cos x + 1 = 0$ (1) ***Chú ý*** : (1) là phương trình bậc 2 với biến $\sin x$ Ta có : $\Delta = (2\cos x + 3)^2 - 8(\cos x + 1) = (2\cos x + 1)^2$ Nghiệm của (1) : $\left[\begin{array}{l}\sin x = \dfrac{2\cos x + 3 + 2\cos x + 1}{4} = \cos x + 1 \\ \sin x = \dfrac{2\cos x + 3 - 2\cos x - 1}{4} = \dfrac{1}{2}\end{array}\right.$ ❖ $\sin x = \dfrac{1}{2} = \sin\dfrac{\pi}{6} \Leftrightarrow \left[\begin{array}{l}x = \dfrac{\pi}{6} + k2\pi \\ x = \dfrac{5\pi}{6} + k2\pi\end{array}\right.\ k \in \mathbb{Z}$	$\left[\begin{array}{l}x = \dfrac{\pi}{6} + k2\pi \\ x = \dfrac{5\pi}{6} + k2\pi \\ x = \dfrac{\pi}{2} + k2\pi \\ x = \pi + k2\pi\end{array}\right.\ k \in \mathbb{Z}$

	❖ $\sin x=\cos x+1 \Leftrightarrow \sin x-\cos x=1 \Leftrightarrow \sin\left(x-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}=\sin\frac{\pi}{4}$	
36 **A2006**	$\frac{2\left(\cos^6 x+\sin^6 x\right)-\sin x\cos x}{\sqrt{2}-2\sin x}=0$ (1) điều kiện : $\sin x\neq\frac{\sqrt{2}}{2}$ (1) $\Leftrightarrow 2\left(\sin^6+\cos^6 x\right)-\sin x\cos x=0$ $\Leftrightarrow 2\left(1-\frac{3\sin^2 2x}{4}\right)-\frac{1}{2}\sin 2x=0$ $\Leftrightarrow 3\sin^2 2x+\sin 2x-4=0 \Leftrightarrow \left[\begin{array}{l}\sin 2x=1\\ \sin 2x=-\frac{4}{3}\end{array}\right.$ ❖ $\sin 2x=1 \Leftrightarrow 2x=\frac{\pi}{2}+k2\pi \Leftrightarrow x=\frac{\pi}{4}+k\pi\ ;k\in\mathbb{Z}$	vì : $\sin x\neq\frac{\sqrt{2}}{2}$ $\Leftrightarrow \left[\begin{array}{l}x\neq\frac{\pi}{4}+k2\pi\\ x\neq\frac{3\pi}{4}+k2\pi\end{array}\right.$ Nghiệm của (1) $x=\frac{5\pi}{4}+k2\pi;k\in\mathbb{Z}$
37 **B2006**	$\cot x+\sin x\left(1+\tan x\tan\frac{x}{2}\right)=4$ (1) Điều kiện : $\begin{cases}\sin 2x\neq 0\\ \cos\frac{x}{2}\neq 0\end{cases}$ Ta có : $1+\tan x.\tan\frac{x}{2}=\frac{1}{\cos x}$ $(1)\Leftrightarrow \frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}=4 \Leftrightarrow \frac{1}{\sin x\cos x}=4$ $\Leftrightarrow 2\sin 2x=1 \Leftrightarrow \sin 2x=\frac{1}{2}=\sin\frac{\pi}{6} \Leftrightarrow \left[\begin{array}{l}2x=\frac{\pi}{6}+k2\pi\\ 2x=\frac{5\pi}{6}+k2\pi\end{array}\right.$	$\left[\begin{array}{l}x=\frac{\pi}{12}+k\pi\\ x=\frac{5\pi}{12}+k\pi\end{array}\right.\ k\in\mathbb{Z}$
38 **D2006**	$\cos 3x+\cos 2x-\cos x-1=0$ $\Leftrightarrow \cos 3x-\cos x+\cos 2x-1=0 \Leftrightarrow 2\sin 2x\sin x+2\sin^2 x=0$ $\Leftrightarrow 2\sin x\left(\sin 2x+\sin x\right)=0 \Leftrightarrow 2\sin x(2\sin x\cos x-\sin x)=0$ $\Leftrightarrow 2\sin^2 x\left(2\cos x-1\right)=0 \Leftrightarrow \left[\begin{array}{l}\sin x=0\\ \cos x=\frac{1}{2}=\cos\frac{\pi}{3}\end{array}\right.$	$\left[\begin{array}{l}x=k\pi\\ x=\pm\frac{\pi}{3}+k2\pi\end{array}\right.\ k\in\mathbb{Z}$
39 **DB 1**	$\cos 3x\cos^3 x-\sin 3x\sin^3 x=\frac{2+3\sqrt{2}}{8}$ (1)	$x=\pm\frac{\pi}{16}+\frac{k\pi}{2}\ ;k\in\mathbb{Z}$

A2006	Ta có $\cos 3x = 4\cos^3 x - 3\cos x \Leftrightarrow \cos^3 x = \frac{1}{4}(\cos 3x + 3\cos x)$ $\sin 3x = 3\sin x - 4\sin^3 x \Leftrightarrow \sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$ $(1) \Leftrightarrow \frac{1}{4}\left[\cos 3x(\cos 3x + 3\cos x) - \sin 3x(3\sin x - \sin 3x)\right] = \frac{2+3\sqrt{2}}{8}$ $\Leftrightarrow \cos 3x(\cos 3x + 3\cos x) - \sin 3x(3\sin x - \sin 3x) = \frac{2+3\sqrt{2}}{2}$ $\Leftrightarrow \cos^2 x + 3\cos 3x\cos x - 3\sin 3x\sin x + \sin^2 3x = 1 + \frac{3\sqrt{2}}{2}$ $\Leftrightarrow 1 + 3(\cos 3x\cos x - \sin 3x\sin x) = 1 + \frac{3\sqrt{2}}{2}$ $\Leftrightarrow \cos 4x = \frac{\sqrt{2}}{2} = \sin\frac{\pi}{4} \Leftrightarrow 4x = \pm\frac{\pi}{4} + k2\pi; k \in \mathbb{Z}$	
40 **DB 2** **A2006**	$2\sin\left(2x - \frac{\pi}{6}\right) + 4\sin x + 1 = 0$ $\Leftrightarrow 2\left[\sin 2x\cos\frac{\pi}{6} - \cos 2x\sin\frac{\pi}{6}\right] + 4\sin x + 1 = 0$ $\Leftrightarrow \sqrt{3}\sin 2x - \cos 2x + 4\sin x + 1 = 0$ $\Leftrightarrow 2\sqrt{3}\sin x\cos x + 4\sin x + 2\sin^2 x = 0$ $\Leftrightarrow 2\sin x\left(\sqrt{3}\cos x + \sin x + 2\right) = 0$ $\Leftrightarrow \left[\begin{array}{l}\sin x = 0\\ \sqrt{3}\cos x + \sin x + 2 = 0\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x = k\pi\\ \cos\left(x - \frac{\pi}{6}\right) = -1\end{array}\right.$	$\left[\begin{array}{l}x = k\pi\\ x = \frac{7\pi}{6} + k2\pi\end{array}\right. k \in \mathbb{Z}$
41 **DB 1** **B2006**	$\left(2\sin^2 x - 1\right)\tan^2 2x + 3\left(2\cos^2 x - 1\right) = 0 \quad (1)$ điều kiện : $\cos 2x \neq 0$ $\Leftrightarrow -\cos 2x.\tan^2 2x + 3\cos 2x = 0$ $(1) \Leftrightarrow \cos 2x\left(\tan^2 2x - 3\right) = 0 \Leftrightarrow \tan^2 2x = 3$ $\Leftrightarrow \left[\begin{array}{l}\tan 2x = \sqrt{3}\\ \tan 2x = -\sqrt{3}\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\tan 2x = \tan\frac{\pi}{3}\\ \tan 2x = \tan\left(-\frac{\pi}{3}\right)\end{array}\right.$	$x = \pm\frac{\pi}{6} + \frac{k\pi}{2}; k \in \mathbb{Z}$

42 **DB 2** **B2006**	$\cos 2x+(1+2\cos x)(\sin x-\cos x)=0$ $\Leftrightarrow (\cos^2 x-\sin^2 x)+(1+2\cos x)(\sin x-\cos x)=0$ $\Leftrightarrow (\cos x-\sin x)(\cos x+\sin x-2\cos x-1)=0$ $\Leftrightarrow \left[\begin{array}{l}\cos x-\sin x=0\\ \sin x-\cos x=1\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\cos\left(x+\frac{\pi}{4}\right)=0\\ \sin\left(x-\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}=\sin\frac{\pi}{4}\end{array}\right.$ $\Leftrightarrow \left[\begin{array}{l}x+\frac{\pi}{4}=\frac{\pi}{2}+k\pi\\ x-\frac{\pi}{4}=\frac{\pi}{4}+k2\pi\\ x-\frac{\pi}{4}=\frac{3\pi}{4}+k2\pi\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{4}+k\pi\\ x=\frac{\pi}{2}+k2\pi \quad k\in\mathbb{Z}\\ x=\pi+k2\pi\end{array}\right.$	$\left[\begin{array}{l}x=\frac{\pi}{4}+k\pi\\ x=\frac{\pi}{2}+k2\pi\text{ ; } k\in\mathbb{Z}\\ x=\pi+k2\pi\end{array}\right.$
43 **DB 1** **D2006**	$\cos^3 x+\sin^3 x+2\sin^2 x=1$ $\Leftrightarrow (\sin x+\cos x)(1-\sin x\cos x)=\cos 2x$ $\Leftrightarrow (\sin x+\cos x)(1-\sin x\cos x)=\cos^2 x-\sin^2 x$ $\Leftrightarrow (\sin x+\cos x)[\sin x-\cos x-\sin x\cos x+1]=0$ $\Leftrightarrow (\sin x+\cos x)\left[(1+\sin x)-\cos x(1+\sin x)\right]=0$ $\Leftrightarrow (\sin x+\cos x)(1+\sin x)(1-\cos x)=0$ $\Leftrightarrow \left[\begin{array}{l}\sin x+\cos x=0\\ \sin x=-1\\ \cos x=1\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\sin\left(x+\frac{\pi}{4}\right)=0\\ x=-\frac{\pi}{2}+k2\pi\\ x=k2\pi\end{array}\right.$	$\left[\begin{array}{l}x=-\frac{\pi}{4}+k\pi\\ x=-\frac{\pi}{2}+k2\pi\,;k\in\mathbb{Z}\\ x=k2\pi\end{array}\right.$
44 **DB 2** **D2006**	$4\sin^3 x+4\sin^2 x+3\sin 2x+6\cos x=0$	$\left[\begin{array}{l}x=-\frac{\pi}{2}+k2\pi\\ x=\pm\frac{2\pi}{3}+k2\pi\end{array}\right. \quad k\in\mathbb{Z}$

	$\Leftrightarrow 4\sin^2 x(\sin x+1)+6\cos x(\sin x+1)=0$ $\Leftrightarrow (\sin x+1)(4\sin^2 x+6\cos x)=0$ $\Leftrightarrow (\sin x+1)\left[4(1-\cos^2 x)+6\cos x\right]=0$ $\Leftrightarrow \left[\begin{array}{l}\sin x=-1\\ 2\cos^2 x-3\cos x-2=0\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\sin x=-1\\ \cos x=2\\ \cos x=-\dfrac{1}{2}\end{array}\right.$	
45 **A2007**	$\left(1+\sin^2 x\right)\cos x+\left(1+\cos^2 x\right)\sin x=1+\sin 2x$ $\Leftrightarrow \cos x+\sin^2 x\cos x+\sin x+\cos^2 x\sin x=(\sin x+\cos x)^2$ $\Leftrightarrow (\sin x+\cos x)+\sin x\cos x(\sin x+\cos x)-(\sin x+\cos x)^2=0$ $\Leftrightarrow (\sin x+\cos x)\left(1+\sin x\cos x-\sin x-\cos x\right)=0$ $\Leftrightarrow (\sin x+\cos x)(1-\sin x)(1-\cos x)=0 \Leftrightarrow \left[\begin{array}{l}\sin x+\cos x=0\\ 1-\sin x=0\\ 1-\cos x=0\end{array}\right.$	$\left[\begin{array}{l}x=-\dfrac{\pi}{4}+k\pi\\ x=\dfrac{\pi}{2}+k2\pi\ ;k\in\mathbb{Z}\\ x=k2\pi\end{array}\right.$
46 **B2007**	$2\sin^2 2x+\sin 7x-1=\sin x$ $\Leftrightarrow \sin 7x-\sin x+2\sin^2 2x-1=0 \Leftrightarrow 2\cos 4x.\sin 3x-\cos 4x=0$ $\Leftrightarrow \cos 4x\left(2\sin 3x-1\right)=0 \Leftrightarrow \left[\begin{array}{l}\cos 4x=0\\ \sin 3x=\dfrac{1}{2}=\sin\dfrac{\pi}{6}\end{array}\right.$	$\left[\begin{array}{l}x=\dfrac{\pi}{8}+\dfrac{k\pi}{4}\\ x=\dfrac{\pi}{18}+\dfrac{k2\pi}{3}\ ;k\in\mathbb{Z}\\ x=\dfrac{5\pi}{18}+\dfrac{k2\pi}{3}\end{array}\right.$
47 **D2007**	$\left(\sin\dfrac{x}{2}+\cos\dfrac{x}{2}\right)^2+\sqrt{3}\cos x=2$ $\Leftrightarrow 1+\sin x+\sqrt{3}\cos x=2 \Leftrightarrow \sin x+\sqrt{3}\cos x=1$ $\Leftrightarrow \sin\left(x+\dfrac{\pi}{3}\right)=\dfrac{1}{2}=\sin\dfrac{\pi}{6} \Leftrightarrow \left[\begin{array}{l}x+\dfrac{\pi}{3}=\dfrac{\pi}{6}+k2\pi\\ x+\dfrac{\pi}{3}=\dfrac{5\pi}{6}+k2\pi\end{array}\right.$	$\left[\begin{array}{l}x=\dfrac{\pi}{2}+k2\pi\\ x=-\dfrac{\pi}{6}+k2\pi\end{array}\right.\ k\in\mathbb{Z}$
48 **DB 1** **A2007**	$\sin 2x+\sin x-\dfrac{1}{2\sin x}-\dfrac{1}{\sin 2x}=2\cot 2x$ (1) điều kiện : $\sin 2x\neq 0$ (1) $\Leftrightarrow \sin^2 2x+\sin 2x\sin x-\cos x-1=2\cos 2x$	$x=\dfrac{\pi}{4}+\dfrac{k\pi}{2};k\in\mathbb{Z}$

	$\Leftrightarrow \sin^2 2x - 1 + \cos x(2\sin^2 x - 1) = 2\cos 2x$ $\Leftrightarrow -\cos^2 2x + \cos 2x.\cos x - 2\cos 2x = 0$ $\Leftrightarrow \cos 2x(\cos 2x + \cos x + 2) = 0$ $\Leftrightarrow \cos 2x(2\cos^2 x + \cos x + 1) = 0 \Leftrightarrow \left[\begin{array}{l}\cos 2x = 0\\ 2\cos^2 x + \cos x + 1 = 0\ (VN)\end{array}\right.$	
49 **DB 2** **A2007**	$2\cos^2 x + 2\sqrt{3}\sin x\cos x + 1 = 3(\sin x + \sqrt{3}\cos x)$ $\Leftrightarrow 2\cos^2 x - 1 + \sqrt{3}\sin 2x + 2 = 3(\sin x + \sqrt{3}\cos x)$ $\Leftrightarrow \cos 2x + \sqrt{3}\sin 2x + 2 = 3(\sin x + \sqrt{3}\cos x)$ $\Leftrightarrow 2 + 2\left(\frac{1}{2}\cos 2x + \frac{\sqrt{3}}{2}\sin 2x\right) = 6\left(\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\right)$ $\Leftrightarrow 2 + 2\cos\left(2x - \frac{\pi}{3}\right) = 6\cos\left(x - \frac{\pi}{6}\right)$ $\Leftrightarrow 1 + \cos 2\left(x - \frac{\pi}{6}\right) = 3\cos\left(x - \frac{\pi}{6}\right)$ $\Leftrightarrow 2\cos^2\left(x - \frac{\pi}{6}\right) - 3\cos\left(x - \frac{\pi}{6}\right) = 0$ $\Leftrightarrow \cos\left(x - \frac{\pi}{6}\right)\left[2\cos\left(x - \frac{\pi}{6}\right) - 3\right] = 0 \Leftrightarrow \left[\begin{array}{l}\cos\left(x - \frac{\pi}{6}\right) = 0\\ \cos\left(x - \frac{\pi}{6}\right) = \frac{3}{2}\end{array}\right.$	$x = \frac{2\pi}{3} + k\pi; k \in \mathbb{Z}$
50 **DB 1** **B2007**	$\sin\left(\frac{5x}{2} - \frac{\pi}{4}\right) - \cos\left(\frac{x}{2} - \frac{\pi}{4}\right) = \sqrt{2}\cos\frac{3x}{2}$ $\Leftrightarrow \sin\left(\frac{5x}{2} - \frac{\pi}{4}\right) - \sin\left[\frac{\pi}{2} + \left(\frac{\pi}{4} - \frac{x}{2}\right)\right] = \sqrt{2}\cos\frac{3x}{2}$ $\Leftrightarrow 2\cos\left(x + \frac{\pi}{4}\right)\sin\left(\frac{3x}{2} - \frac{\pi}{2}\right) = \sqrt{2}\cos\frac{3x}{2}$ $\Leftrightarrow -2\cos\left(x + \frac{\pi}{4}\right)\cos\frac{3x}{2} = \sqrt{2}\cos\frac{3x}{2}$ $\Leftrightarrow \cos\frac{3x}{2}\left[\sqrt{2} + 2\cos\left(x + \frac{\pi}{4}\right)\right] = 0 \Leftrightarrow \left[\begin{array}{l}\cos\frac{3x}{2} = 0\\ \cos\left(x + \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}\end{array}\right.$	$\left[\begin{array}{l}x = \frac{\pi}{3} + \frac{k2\pi}{3}\\ x = \frac{\pi}{2} + k2\pi\ ; k \in \mathbb{Z}\\ x = \pi + k2\pi\end{array}\right.$

51 **DB 2** **B2007**	$\frac{\sin 2x}{\cos x}+\frac{\cos 2x}{\sin x}=\tan x-\cot x \quad (1)$ điều kiện : $\sin 2x\neq 0$ $(1)\Leftrightarrow \frac{\cos 2x.\cos x+\sin 2x.\sin x}{\sin x\cos x}=\frac{\sin x}{\cos x}-\frac{\cos x}{\sin x}$ $\Leftrightarrow \frac{\cos x}{\sin x\cos x}=\frac{\sin^2 x-\cos^2 x}{\sin x\cos x}\Leftrightarrow \cos x+\cos 2x=0$ $\Leftrightarrow 2\cos^2 x+\cos x-1=0\Leftrightarrow \left[\begin{array}{l}\cos x=-1 \ (L)\\ \cos x=\frac{1}{2}\end{array}\right.$	$x=\pm\frac{\pi}{3}+k2\pi;k\in\mathbb{Z}$
52 **DB1** **D2007**	$2\sqrt{2}\sin\left(x-\frac{\pi}{12}\right)\cos x=1$ $\Leftrightarrow \sqrt{2}\left[\sin\left(2x-\frac{\pi}{12}\right)-\sin\frac{\pi}{12}\right]=1\Leftrightarrow \sin\left(2x-\frac{\pi}{12}\right)-\sin\frac{\pi}{12}=\frac{1}{\sqrt{2}}$ $\Leftrightarrow \sin\left(x-\frac{\pi}{12}\right)=\sin\frac{\pi}{4}+\sin\frac{\pi}{12}=2\sin\frac{\pi}{6}\cos\frac{\pi}{12}$ $\Leftrightarrow \sin\left(x-\frac{\pi}{12}\right)=\cos\frac{\pi}{12}=\cos\left(\frac{\pi}{2}-\frac{5\pi}{12}\right)=\sin\frac{5\pi}{12}$ $\Leftrightarrow \left[\begin{array}{l}2x-\frac{\pi}{12}=\frac{5\pi}{12}+k2\pi\\ 2x-\frac{\pi}{12}=\frac{7\pi}{12}+k2\pi\end{array}\right.$	$\left[\begin{array}{l}x=\frac{\pi}{4}+k\pi\\ x=\frac{\pi}{3}+k\pi\end{array}\right. k\in\mathbb{Z}$
53 **DB1** **D2007**	$(1-\tan x)(1+\sin 2x)=1+\tan x \quad (1)$ điều kiện : $\cos x\neq 0$ $(1)\Leftrightarrow \frac{\cos x-\sin x}{\cos x}.(\sin x+\cos x)^2=\frac{\sin x+\cos x}{\cos x}$ $\Leftrightarrow (\cos x-\sin x)(\sin x+\cos x)^2=\cos x+\sin x$ $\Leftrightarrow (\cos x+\sin x)\big((\cos x-\sin x)(\cos x+\sin x)-1\big)=0$ $\Leftrightarrow (\cos x+\sin x)(\cos^2 x-\sin^2 x-1)=0$ $\Leftrightarrow (\cos x+\sin x)(\cos 2x-1)=0\Leftrightarrow \left[\begin{array}{l}\cos x+\sin x=0\\ \cos 2x=1\end{array}\right.$ $\Leftrightarrow \left[\begin{array}{l}\cos\left(x-\frac{\pi}{4}\right)=0\\ \cos 2x=1\end{array}\right.\Leftrightarrow \left[\begin{array}{l}x-\frac{\pi}{4}=\frac{\pi}{2}+k\pi\\ 2x=k\pi\end{array}\right.$	$\left[\begin{array}{l}x=-\frac{\pi}{4}+k\pi\\ x=k\pi\end{array}\right. k\in\mathbb{Z}$

54 **A2008**	$\dfrac{1}{\sin x}+\dfrac{1}{\sin\left(x-\dfrac{3\pi}{2}\right)}=4\sin\left(\dfrac{7\pi}{4}-x\right)$ (1) Điều kiện : $\sin x\neq 0$ và $\sin\left(x-\dfrac{3\pi}{2}\right)\neq 0$ $(1)\Leftrightarrow\dfrac{1}{\sin x}+\dfrac{1}{\cos x}=-2\sqrt{2}(\sin x+\cos x)$ ***Chú ý*** : $\sin\left(x-\dfrac{3\pi}{2}\right)=\cos x$ $\sin\left(\dfrac{7\pi}{4}-x\right)=-\sin\left(x+\dfrac{\pi}{4}\right)=-\dfrac{1}{\sqrt{2}}\left(\sin x+\cos x\right)$ $(1)\Leftrightarrow\dfrac{1}{\sin x}+\dfrac{1}{\cos x}=-2\sqrt{2}(\sin x+\cos x)$ $\Leftrightarrow\dfrac{\sin x+\cos x}{\sin x\cos x}=-2\sqrt{2}(\sin x+\cos x)$ $\Leftrightarrow(\sin x+\cos x)\left(\dfrac{1}{\sin x\cos x}+2\sqrt{2}\right)=0$ $\Leftrightarrow 2\left(\sin x+\cos x\right)\left(\dfrac{1+\sqrt{2}\sin 2x}{\sin 2x}\right)=0\Leftrightarrow\left[\begin{array}{l}\sin x+\cos x=0\\ \sin 2x=-\dfrac{\sqrt{2}}{2}=\sin\left(-\dfrac{\pi}{4}\right)\end{array}\right.$	$\left[\begin{array}{l}x=-\dfrac{\pi}{4}+k\pi\\ x=-\dfrac{\pi}{8}+k\pi\,;k\in\mathbb{Z}\\ x=\dfrac{5\pi}{8}+k\pi\end{array}\right.$
55 **B2008**	$\sin^3 x-\sqrt{3}\cos^3 x=\sin x\cos^2 x-\sqrt{3}\sin^2 x\cos x$ $\Leftrightarrow\sin x(\cos^2 x-\sin^2 x)+\sqrt{3}\cos x(\cos^2 x-\sin^2 x)=0$ $\Leftrightarrow\cos 2x(\sin x+\sqrt{3}\cos x)=0$ $\Leftrightarrow\left[\begin{array}{l}\cos 2x=0\\ \sin x+\sqrt{3}\cos x=0\end{array}\right.\Leftrightarrow\left[\begin{array}{l}\cos 2x=0\\ \sin\left(x+\dfrac{\pi}{3}\right)=0\end{array}\right.$	$\left[\begin{array}{l}x=\dfrac{\pi}{4}+\dfrac{k\pi}{2}\\ x=-\dfrac{\pi}{3}+k\pi\end{array}\right.\quad k\in\mathbb{Z}$
56 **D2008**	$2\sin x\left(1+\cos 2x\right)+\sin 2x=1+2\cos x$ $\Leftrightarrow 4\sin x\cos^2 x+\sin 2x=1+2\cos x$ $\Leftrightarrow\sin 2x(2\cos x+1)-(1+2\cos x)=0\Leftrightarrow(2\cos x+1)(\sin 2x-1)=0$ $\Leftrightarrow\left[\begin{array}{l}2\cos x=-1\\ \sin 2x=1\end{array}\right.\Leftrightarrow\left[\begin{array}{l}\cos x=-\dfrac{1}{2}\\ \sin 2x=1\end{array}\right.$	$\left[\begin{array}{l}x=\pm\dfrac{2\pi}{3}+k2\pi\\ x=\dfrac{\pi}{4}+k\pi\end{array}\right.\quad k\in\mathbb{Z}$

57 **CĐ** **2008**	$\sin 3x-\sqrt{3}\cos 3x=2\sin 2x$ $\Leftrightarrow \frac{1}{2}\sin 3x-\frac{\sqrt{3}}{2}\cos 3x=\sin 2x$ $\Leftrightarrow \sin\left(3x-\frac{\pi}{3}\right)=\sin 2x \Leftrightarrow \left[\begin{array}{l} 3x-\frac{\pi}{3}=2x+k2\pi \\ 3x-\frac{\pi}{3}=\pi-2x+k2\pi \end{array}\right. \quad k\in\mathbb{Z}$	$\left[\begin{array}{l} x=\frac{\pi}{3}+k2\pi \\ x=\frac{4\pi}{15}+\frac{k2\pi}{5} \end{array}\right. \quad k\in\mathbb{Z}$
58 **DB 1** **A2008**	$\tan x=\cot x+4\cos^2 2x \quad (1)$ điều kiện : $\sin 2x\neq 0$ $(1)\Leftrightarrow \frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}+4\cos^2 2x=0$ $\Leftrightarrow \cos 2x+2\cos^2 2x\sin 2x=0 \Leftrightarrow \cos 2x+\sin 4x.\cos 2x=0$ $\Leftrightarrow \cos 2x(1+\sin 4x)=0 \Leftrightarrow \left[\begin{array}{l} \cos 2x=0 \\ \sin 4x=-1 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} 2x=\frac{\pi}{2}+k\pi \\ 4x=-\frac{\pi}{4}+k2\pi \end{array}\right.$	$\left[\begin{array}{l} x=\frac{\pi}{4}+\frac{k\pi}{2} \\ x=-\frac{\pi}{8}+\frac{k\pi}{2} \end{array}\right. \quad k\in\mathbb{Z}$
59 **DB 2** **A2008**	$\sin\left(2x-\frac{\pi}{4}\right)=\sin\left(x-\frac{\pi}{4}\right)+\frac{\sqrt{2}}{2}$ $\Leftrightarrow \frac{1}{\sqrt{2}}(\sin 2x-\cos 2x)=\frac{1}{\sqrt{2}}(\sin x-\cos x+1)$ $\Leftrightarrow \sin 2x-\sin x-(1+\cos 2x)+\cos x=0$ $\Leftrightarrow \sin x(2\cos x-1)-2\cos^2 x+\cos x=0$ $\Leftrightarrow \sin x(2\cos x-1)-\cos x(2\cos x-1)=0$ $\Leftrightarrow (2\cos x-1)(\sin x-\cos x)=0$ $\Leftrightarrow \left[\begin{array}{l} 2\cos x-1=0 \\ \sin x-\cos x=0 \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \cos x=\frac{1}{2} \\ \sin\left(x-\frac{\pi}{4}\right)=0 \end{array}\right.$	$\left[\begin{array}{l} x=\pm\frac{\pi}{3}+k2\pi \\ x=\frac{\pi}{4}+k\pi \end{array}\right. \quad k\in\mathbb{Z}$
60 **DB 1** **B2008**	$2\sin\left(x+\frac{\pi}{3}\right)-\sin\left(2x-\frac{\pi}{6}\right)=\frac{1}{2}$	$\left[\begin{array}{l} x=-\frac{\pi}{3}+k\pi \\ x=\frac{\pi}{2}+k2\pi \end{array}\right. \quad k\in\mathbb{Z}$

	$\Leftrightarrow \sin x+\sqrt{3}\cos x-\left(\frac{\sqrt{3}}{2}\sin 2x-\frac{1}{2}\cos 2x\right)=\frac{1}{2}$ $\Leftrightarrow \sin x+\sqrt{3}\cos x-\sqrt{3}\sin x\cos x+\frac{1-2\sin^2 x}{2}=\frac{1}{2}$ $\Leftrightarrow \sqrt{3}\cos x\left(1-\sin x\right)+\sin x\left(1-\sin x\right)=0$ $\Leftrightarrow (1-\sin x)(\sqrt{3}\cos x+\sin x)=0$ $\Leftrightarrow \left[\begin{array}{l}\sin x=1\\ \sqrt{3}\cos x+\sin x=0\end{array}\right. \Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{2}+k2\pi\\ \sin\left(x+\frac{\pi}{3}\right)=0\end{array}\right.$	
61 **DB 2** **B2008**	$3\sin x+\cos 2x+\sin 2x=4\sin x\cos^2\frac{x}{2}$ $\Leftrightarrow 3\sin x+\cos 2x+\sin 2x=4\sin x\left(\frac{1+\cos x}{2}\right)$ $\Leftrightarrow 3\sin x+\cos 2x+\sin 2x=2\sin x+\sin 2x$ $\Leftrightarrow \cos 2x+\sin x=0 \Leftrightarrow 2\sin^2 x-\sin x-1=0$ $\Leftrightarrow \left[\begin{array}{l}\sin x=1\\ \sin x=-\frac{1}{2}=\sin\left(-\frac{\pi}{6}\right)\end{array}\right.$	$\left[\begin{array}{l}x=\frac{\pi}{2}+k2\pi\\ x=-\frac{\pi}{6}+k2\pi\,;k\in\mathbb{Z}\\ x=\frac{7\pi}{6}+k2\pi\end{array}\right.$
62 **DB 1** **D2008**	$4\left(\sin^4 x+\cos^4 x\right)+\cos 4x+\sin 2x=0$ $\Leftrightarrow 4\left(1-\frac{\sin^2 2x}{2}\right)+1-2\sin^2 2x+\sin 2x=0$ $\Leftrightarrow 4\sin^2 2x-\sin 2x-5=0 \Leftrightarrow \left[\begin{array}{l}\sin 2x=-1\\ \sin 2x=\frac{5}{4}(L)\end{array}\right.$	$x=-\frac{\pi}{4}+k\pi;k\in\mathbb{Z}$
63 **DB 2** **D2008**	$\frac{\tan^2 x+\tan x}{\tan^2 x+1}=\frac{\sqrt{2}}{2}\sin\left(x+\frac{\pi}{4}\right)$ (1) điều kiện : $\cos x\neq 0$ $(1)\Leftrightarrow \frac{\tan^2 x+\tan x}{\tan^2 x+1}=\frac{1}{2}\left(\sin x+\cos x\right)$	$\left[\begin{array}{l}x=-\frac{\pi}{4}+k\pi\\ x=\frac{\pi}{6}+k2\pi\quad;k\in\mathbb{Z}\\ x=\frac{5\pi}{6}+k2\pi\end{array}\right.$

	$\Leftrightarrow 2\cos^2 x\left(\tan^2 x+\tan x\right)=\sin x+\cos x$ $\Leftrightarrow 2\cos^2 x\left(\dfrac{\sin^2 x+\sin x\cos x}{\cos^2 x}\right)=\sin x+\cos x$ $\Leftrightarrow 2\sin x\left(\sin x+\cos x\right)-\left(\sin x+\cos x\right)=0$ $\Leftrightarrow \left(\sin x+\cos x\right)\left(2\sin x-1\right)=0$ $\Leftrightarrow \left[\begin{array}{l}\sin x+\cos x=0\\ 2\sin x=1\end{array}\right. \Leftrightarrow \left[\begin{array}{l}\sin\left(x+\dfrac{\pi}{4}\right)=0\\ \sin x=\dfrac{1}{2}=\cos\dfrac{\pi}{6}\end{array}\right.$	
64 **A2009**	$\dfrac{(1-2\sin x)\cos x}{(1+2\sin x)(1-\sin x)}=\sqrt{3}$ (1) điều kiện : $\left\{\begin{array}{l}\sin x\neq 1\\ \sin x\neq -\dfrac{1}{2}\end{array}\right.$ $(1)\Leftrightarrow \left(1-2\sin x\right)\cos x=\sqrt{3}(1+\sin 2x)(1-\sin x)$ $\Leftrightarrow \cos x-\sin 2x=\sqrt{3}\left(1+\sin x-2\sin^2 x\right)$ $\Leftrightarrow \cos x-\sin 2x=\sqrt{3}\left(\cos 2x+\sin x\right)$ $\Leftrightarrow \cos x-\sqrt{3}\sin x=\sin 2x+\sqrt{3}\cos 2x$ $\Leftrightarrow \dfrac{1}{2}\cos x-\dfrac{\sqrt{3}}{2}\sin x=\dfrac{1}{2}\sin 2x+\dfrac{\sqrt{3}}{2}\cos 2x$ $\Leftrightarrow \cos\left(x+\dfrac{\pi}{3}\right)=\cos\left(2x-\dfrac{\pi}{6}\right)\Leftrightarrow \left[\begin{array}{l}2x-\dfrac{\pi}{6}=x+\dfrac{\pi}{3}+k2\pi\\ 2x-\dfrac{\pi}{6}=-x-\dfrac{\pi}{3}+k2\pi\end{array}\right.$ $\Leftrightarrow \left[\begin{array}{l}x=\dfrac{\pi}{2}+k2\pi\\ x=-\dfrac{\pi}{18}+\dfrac{k2\pi}{3}\end{array}\right. k\in\mathbb{Z}$	$x=-\dfrac{\pi}{18}+\dfrac{k2\pi}{3}$ **;** $k\in\mathbb{Z}$

65 **B2009**	$\sin x + \cos x \sin 2x + \sqrt{3}\cos 3x = 2\left(\cos 4x + \sin^3 x\right)$ $\Leftrightarrow \sin x\left(1-2\sin^2 x\right) + \cos x \sin 2x + \sqrt{3}\cos 3x = 2\cos 4x$ $\Leftrightarrow \sin x \cos 2x + \cos x \sin 2x + \sqrt{3}\cos 3x = 2\cos 4x$ $\Leftrightarrow \sin 3x + \sqrt{3}\cos 3x = 2\cos 4x \Leftrightarrow \frac{1}{2}\sin 3x + \frac{\sqrt{3}}{2}\cos 3x = \cos 4x$ $\Leftrightarrow \cos\left(3x - \frac{\pi}{6}\right) = \cos 4x \Leftrightarrow \left[\begin{array}{l} 4x = 3x - \frac{\pi}{6} + k2\pi \\ 4x = -3x + \frac{\pi}{6} + k2\pi \end{array}\right.$	$\left[\begin{array}{l} x = -\frac{\pi}{6} + k2\pi \\ x = \frac{\pi}{42} + \frac{k2\pi}{7} \end{array}\right. k \in \mathbb{Z}$
66 **D2009**	$\sqrt{3}\cos 5x - 2\sin 3x \cos 2x - \sin x = 0$ $\Leftrightarrow \sqrt{3}\cos 5x - \left(\sin 5x + \sin x\right) - \sin x = 0$ $\Leftrightarrow \sqrt{3}\cos 5x - \sin 5x = 2\sin x \Leftrightarrow \frac{\sqrt{3}}{2}\cos 5x - \frac{1}{2}\sin 5x = \sin x$ $\Leftrightarrow \sin\left(\frac{\pi}{3} - 5x\right) = \sin x \Leftrightarrow \left[\begin{array}{l} x = \frac{\pi}{3} - 5x + k2\pi \\ x = \frac{2\pi}{3} + 5x + k2\pi \end{array}\right. \Leftrightarrow \left[\begin{array}{l} 6x = \frac{\pi}{3} + k2\pi \\ -4x = \frac{2\pi}{3} + k2\pi \end{array}\right.$	$\left[\begin{array}{l} x = -\frac{\pi}{6} - \frac{k\pi}{2} \\ x = \frac{\pi}{18} + \frac{k\pi}{3} \end{array}\right. k \in \mathbb{Z}$
67 **CĐ** **2009**	$(1+2\sin x)^2 \cos x = 1 + \sin x + \cos x$ $\Leftrightarrow (1 + 4\sin x + 4\sin^2 x)\cos x = 1 + \sin x + \cos x$ $\Leftrightarrow \cos x + 2\sin 2x + 4\sin^2 x \cos x - 1 - \sin x - \cos x = 0$ $\Leftrightarrow \left(2\sin 2x - 1\right) + \sin x\left(2\sin 2x - 1\right) = 0 \Leftrightarrow \left[\begin{array}{l} \sin 2x = \frac{1}{2} = \sin\frac{\pi}{6} \\ \sin x = -1 \end{array}\right.$	$\left[\begin{array}{l} x = -\frac{\pi}{2} + k2\pi \\ x = \frac{\pi}{12} + k\pi \\ x = \frac{5\pi}{12} + k\pi \end{array}\right. ;k \in \mathbb{Z}$